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Investigating the Performance of Budget Allocation Rules: A Monte Carlo Study

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Report Summary

The budget allocation process is one of the marketing manager's most important tasks. With a portfolio of products and various marketing activities with dynamic impact on future sales, the profit maximization problem is highly complex. While there are various optimization approaches, surveys among managers consistently reveal that they prefer simple allocation rules such as percentage-of-sales rules. Surprisingly, given the high managerial importance of budget optimization, not much is known about how different methods perform across varying firm and market conditions.

In an experimental simulation study, Marc Fischer, Nils Wagner, and Sönke Albers investigate the performance of four methods in allocating a fixed marketing budget across products and marketing activities to maximize discounted portfolio profit over a five-period planning horizon. These methods include a naïve allocation (an equal distribution across all products and activities, ignoring heterogeneity in the product portfolio), a percentage-of-sales rule (total budget is allocated proportional to the previous year's sales), an attractiveness heuristic (which incorporates information on the profit improvement potential of allocating a higher budget to the unit), and a numerical optimization method (which generates optimal budgets for a specified problem).

They apply the allocation rules in a multitude of systematically varied scenarios in order to analyze and compare their performance as well as their sensitivity to different characteristics of the market environment. Their evaluation is based on the profit gained by application of the respective allocation rule compared to the optimal solution. Since it would be unrealistic to assume that managers know the true values of unobservable demand parameters, they analyze the sensitivity of the different rules by imposing an estimation error, which affects the parameters of interest.

Their study reveals important insights into the performance characteristics of the methods. A theoretically founded heuristic rule, such as the attractiveness heuristic, converges quickly to the optimum and is reliable even under extreme conditions. An exact method such as numerical optimization is optimal by definition. However, if true demand parameters are not known but estimated with an error, numerical optimization no longer produces optimal results. In fact, its suboptimality is considerably higher than that of the attractiveness heuristic. The percentage-of-sales rule produces better allocation results over time and outperforms numerical optimization in extreme scenarios with noisy demand parameters. The heterogeneity of marketing responsiveness and product age have the greatest influence on the performance of an allocation method.

These insights have important implications for managers, most notably that an exact method such as numerical optimization is inferior to a decision heuristic if it is applied under the realistic assumption that true demand parameters are unknown. Overall, theoretically derived heuristics such as the attractiveness heuristic demonstrate a remarkably reliable performance.

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INVESTIGATING THE PERFORMANCE OF BUDGET ALLOCATION RULES: A MONTE CARLO STUDY

Introduction

Setting the right marketing budget has been a key research problem and a top challenge to management for a long time. For companies that market a portfolio of products and use different marketing activities, it involves finding the optimal total marketing budget and its optimal allocation across allocation units such as products and activities. Theoretical and empirical research (e.g., Fischer et al. 2011; Mantrala, Sinha, and Zoltners 1992) shows that solving the second problem, the optimal allocation of a marketing budget, is much more important. Better allocation results in profit gains between 40% and 80%, whereas the optimization of the overall budget level improves profit only by 3-5% (e.g., Doyle and Saunders 1990; Mantrala, Sinha, and Zoltners 1992). Ideally, both problems are solved simultaneously. Theoretically, this can be achieved but at the cost of higher complexity and efforts to guarantee the uniqueness of an optimal solution. Practically, companies separate these problems. Top management usually determines the overall marketing budget for the next fiscal year first. This budget is then allocated across country units, products, marketing activities, etc. (Perrey and Spillecke 2011).

Consequently, academics have developed methods and algorithms to solve complex allocation problems under a restricted marketing budget (e.g., Lodish 1971; Mantrala 2002). Unfortunately, these optimization approaches often rely on numerical optimization techniques that are not used by managers. In fact, surveys among managers consistently reveal that they prefer simple allocation rules such as the percentage-of-sales rule. But these practitioner rules are supposed to lead to allocation results that are rather far away from the optimum. Therefore research started to find a way out of this dilemma by suggesting new heuristics that are derived from theory and also accepted by managers (e.g., Fischer et al. 2011). In real-world applications, these heuristics seem to lead to large profit gains. However, this performance may be due to the specific conditions in an application and does not necessarily generalize to other situations.

To summarize, management can choose among several methods to solve the important marketing budget allocation problem. All these methods probably have their advantages and disadvantages in terms of optimality, practical applicability, etc. Surprisingly, despite the high managerial relevance of budget optimization, we do not know how well the methods perform relative to each other across varying market and firm conditions. Given that practitioner rules and decision heuristics do not guarantee the optimal solution, the question is how close they come to the optimum and how they perform over time when they are repeatedly applied.

We address this research gap by setting up a simulation experiment in which we analyze the performance of four budgeting rules, manipulate all relevant factors, and further examine the effect of estimation error. The advantage of simulation experiments is that the true parameter values are known, their true relation is given and all characteristics of interest can be fully controlled. As a consequence the optimal solution is known and the suboptimality of applying the various allocation methods can be assessed. This is not possible with real-life data, simply because the real parameters and models are not known.

Specifically, we consider the naïve budgeting approach of an equal distribution, the most common practitioner rule to allocate the budget proportional to product sales, the allocation heuristic by Fischer, Albers, Wagner, and Frie (2011), denoted hereafter as FAWF, and a numerical optimization solution. Our experimental design considers dynamic effects and manipulates all factors and functions that are incorporated into the dynamic profit maximization problem. This enables us to derive generalizable results and to analyze the impact of several factors on the performance of each of the considered budgeting methods. As a further aspect we create more realistic scenarios by imposing estimation errors on unobservable demand parameters. An analysis of the sensitivity to estimation error provides insights into how the performance of allocation rules changes if their estimated parameters are exposed to noisiness. In particular, our simulation study seeks to answer the following research questions:

- How do the allocation methods perform relative to the optimal solution? Are they close to being "optimal"?
- How do the allocation methods perform over time, i.e. by being subsequently applied? Do they converge to the optimal solution?
- How reliable are the methods under extreme conditions?
- If the allocation methods include unobservable demand parameters that need to be estimated: How strongly are these methods influenced by estimation errors?
- Which are the most important factors that influence the performance (and the convergence properties) of the allocation rules?

We follow prior simulation studies in marketing research to develop a Monte Carlo design (e.g., Andrews, Ainslie and Currim 2008). Note that these studies have much in common with simulation studies in statistics. They usually analyze the performance of empirical methods to describe and predict demand behavior such as brand choice under different conditions. Typical performance measures are the recoverability of behavioral parameters and predictive accuracy. In contrast, our study shares features of simulation studies in operations research. The objective is to study the optimality of firm behavior, i.e. to set the "right" marketing budget across allocation units, by using different decision rules. As a consequence, the deviation from profit maximum and the speed of converging to that optimum are the relevant performance measures.

The rest of the paper is organized as follows. We continue in section 2 by describing the allocation decision problem and summarizing prior literature on marketing resource allocation. In section 3, we explain the design of the simulation study. Sections 4 and 5 discuss the results of the simulation experiment and identify the drivers of the performance (and the convergence properties) of the allocation rules. We close with limitations and suggestions for future research.

Allocating Marketing Resources

Description of the allocation problem

Following Fischer et al. (2011), we consider the general allocation decision problem of a multi-product, multi-activity firm that wishes to maximize the net present value Π of its product portfolio over a planning period T, e.g. five years, by effectively allocating a fixed marketing budget R. The firm faces the constrained dynamic profit maximization as formulated in equations (1.1)-(1.4):

$$\max_{S_{it}} \Pi = \underbrace{\sum_{t=0}^{T} \underbrace{\frac{1}{(1+r)^t}}_{\text{Discounting}} \left\{ \left[\sum_{i \in I} \underbrace{(p_{it} - c_{it})}_{\text{Profit contribution}} \cdot \underbrace{q_{it}\left(ET_{it} + t, \mathbf{S}_{it}, \mathbf{Z}_{it}\right)}_{\text{Unit sales}} \right] - \underbrace{\sum_{i \in I} \sum_{n \in N_i} x_{int}}_{\text{Marketing expenditures}} \right\}}_{\text{Discounted net value of product portfolio}} \tag{1.1}$$

subject to $R_t = \sum_{i \in I} \sum_{n \in N_i} x_{int}$, with $R_t - R_{t-1} = 0$ (budget constraint), (1.2)

$S_{int} - S_{int-1} = -\zeta_{in} S_{int-1} + x_{int}$, with $x_{int} \geq 0$ (state variable equation), (1.3)

$S_{int} \geq 0$, $S_{int=0}$, and $S_{int=T} = S_{inT}$ (boundary conditions), (1.4)

where t is the time period with planning horizon T. The product is denoted by i with the index set I. The marketing budget can be spent on any marketing activity that uses financial resources, such as advertising, sales force, distributional efforts, etc. We denote the type of marketing activity or spending category, respectively, by n. N_i is the associated index set that may vary across products. The discount rate is denoted by r, $0 < r < \infty$. Price p minus marginal cost c defines the profit contribution per unit sold. Unit sales q are determined by a function which is influenced by the elapsed time since launch of the product ET, the marketing stock **S**, which is a

N_i-dimensional row vector summarizing the activity-specific marketing stocks for product i and a row vector of other variables **Z** (e.g., competitive marketing spending). To reflect the long-term impact of marketing spending, the marketing stock S follows a dynamic process that satisfies the difference equation (1.3), where x denotes marketing expenditures and ζ is the depreciation rate of the marketing stock (Nerlove and Arrow 1962). In addition, the sales function accounts for life-cycle effects by including a life cycle function, whose growth parameters are influenced by marketing investments. In our experimental setting, we compare different demand and growth functions. Boundary conditions (1.4) define the initial and end values for marketing stocks and ensure they are nonnegative. Finally, the total marketing budget R is fixed and equals the sum of all activity-specific marketing expenditures x. It is assumed to be constant over the planning horizon. Top management, however, may decide to adjust the level in future planning cycles.

The decision problem is general and poses several challenges for finding the optimal allocation solution. It is a multi-period decision problem that requires balancing investments in a portfolio of products that are in different stages of their life cycle. Young products should receive sufficient resources even though their current profit contribution may be negative. Marketing expenditures are assumed to have carryover effects. Finally, we assume that firms compete with each other and set their marketing budgets in a manner consistent with Nash behavior.

Research on marketing resource allocation

Marketing resource allocation problems have attracted a considerable amount of research. Wagner and Fischer (2012) review descriptive studies that seek to understand how managers actually set marketing budgets. This research stream is largely based on manager surveys. From the analysis of 26 survey studies, the authors find that managers basically use simple heuristics and hardly consider exact methods such as numerical optimization approaches. In general, managers apply more than one heuristic, whereas the percentage-of-sales rule dominates with 80% of entries and is followed by the objective-and-task rule with ca. 50% of entries. At least 15% of managers mentioned that they set budgets based on previous year's budget.

Mantrala (2002) gives a comprehensive overview of the normative literature and suggests that models can be grouped into normative-theoretical models, which produce generalizable theoretical results, and decision models that are designed to solve specific problems.

The literature on normative-theoretical allocation models is very sparse. An early contribution is the seminal work by Dorfman and Steiner (1954) who establish optimal conditions for marketing mix allocations. Mantrala, Sinha and Zoltners (1992) develop a normative-theoretical model to analyze the sensitivity of profit to allocation errors for a product that faces different levels of sales responsiveness in two submarkets. They investigate three types of sales response functions including a scenario with uncertain demand. Their analysis reveals that simple allocation rules such as a sales-proportional budget allocation may lead to substantial investment errors under various market scenarios. Clearly, this raises doubts about the usefulness of practitioner rules. The study, however, has its limitations. The analysis is focused on a single period and a single product. Hence, dynamics and heterogeneity of products in a portfolio that differ in age are not considered. So, it is unclear whether the repeated application of a heuristic eventually converges to the optimal steady-state solution as was argued by Welam (1982).

The literature on decision models for allocation problems is much broader. By purpose, these models were designed to solve specific decision problems. Consistent with the setup of our decision problem, we discuss only models that deal with allocations *across products* of a portfolio. Doyle and Saunders (1990) develop a model that produces the optimal allocation of an advertising budget across media and product categories of a department store. Lodish (1971) introduced an integer-programming approach known as CALLPLAN to generate the optimal allocation of sales force efforts across products. Bultez and Naert (1988) developed S.H.A.R.P., a decision model that allocates a limited shelf space across SKUs. Silva-Risso, Bucklin, and Morrison (1999) suggested a decision support system that optimizes manufacturers' trade promotion calendars across brand SKUs. Several other models have been suggested that extend these models or focus on similar problems (see Mantrala 2002).

The vast majority of these models use a numerical optimization algorithm to solve for the allocation solution. Interestingly, almost all models optimize current period profits and thus ignore dynamic effects. Finally, we note that no allocation model incorporates the dynamics that result from competitive interaction.

A decision model that assumes Nash competition and allocates a fixed budget across communication spend categories, countries, and products over a planning horizon to maximize multi-period profits was recently suggested by Fischer et al. (2011). The authors, however, do not suggest a numerical optimization routine but derive an easy-to-implement heuristic from the first-order conditions of the dynamic optimization problem.

Allocation rules investigated in the experimental simulation study

Following the observation that marketing managers predominantly use simple rules or heuristics, respectively, to allocate the marketing budget, we investigate the performance of three such rules. We compare their performance with the results of an exact numerical optimization method. Specifically, we consider a naïve allocation that distributes the budget equally across the product portfolio and the most frequently used practitioner rule, the percentage-of-sales rule.

Further, we analyze the performance of the heuristic proposed by FAWF (2011) that is based on the optimality conditions of the dynamic profit maximization problem.[1]

Naïve allocation: equal distribution. The most naïve approach is an equal distribution across all products and activities, which ignores the heterogeneity of the product portfolio. The budget allocation is easily obtained by dividing the total budget by the number of allocation units. An allocation unit in our problem formulation (1.1)-(1.4) is defined as a specific marketing activity for a specific product. The naïve allocation serves as a lower-bound benchmark to compare with other allocation approaches.

Percentage-of-sales rule. The percentage-of-sales rule proposes to set the marketing budget as a specific percentage of previous year's sales. Applying this rule to a portfolio implies that the total budget is allocated proportional to product sales, i.e. products with a greater sales level get a larger proportion of the marketing budget and vice versa.[2] Because the rule makes no recommendation for the allocation of the product budget across marketing activities we simply assume equal shares for them.

Attractiveness allocation heuristic by FAWF (2011). FAWF (2011) propose to allocate the budget for marketing activity n of product i proportional to its allocation weight w:

$$\tilde{x}_{int}^{FAWF} = \frac{\tilde{w}_{int}}{\sum_{j \in I} \sum_{m \in N_j} \tilde{w}_{jmt}} R_t, \forall i \in I, n \in N_i, t \in [0, T] \tag{2.1}$$

$$\text{with } \tilde{w}_{int} = \underbrace{\varepsilon_{in,t-1} / (r + 1 - \delta_{in})}_{\text{Long-term marketing effectiveness}} \cdot \underbrace{cm_i \cdot RV_{i,t-1}}_{\text{Profit contribution}} \cdot \underbrace{\rho_{it}}_{\text{Growth potential}}, \tag{2.2}$$

where

$\tilde{x}_{int}^{FAWF}$: Marketing budget for marketing activity n and product i in period t;
$\tilde{w}_{int}$: Heuristic allocation weight for marketing activity n and product i in period t;

[1] Note that the naïve allocation implies constant product budgets under a fixed total budget. It is therefore a
[2] Note that while the percentage applied across products is the same within a year it does not need to be constant across years.

R_t : Total budget to be allocated in period t;
r : Discount rate (capital cost of firm, strategic business unit, etc.);
δ_{in} : Carryover coefficient of marketing activity n for product i (with $\delta_{in} = 1- \zeta_{in}$);
$\varepsilon_{in,t-1}$: Short-term sales elasticity with respect to product i's marketing expenditures on activity n available from last year;
cm_i : (Percentage) contribution margin for product i;
$RV_{i,t-1}$: Revenue level of product i available from last year;
ρ_{it} : Multiplier to measure the growth potential of product i in period t;
i = 1, 2, …, I_k (index for products);
n = 1, 2, …, N_i (index for marketing activities); and
t = 1, 2, …, T (index for periods).

This allocation heuristic is directly derived from the optimality conditions that need to be satisfied for solving the dynamic optimization problem (for details see FAWF 2011). Basically, the rule teaches to allocate the total budget proportional to the relative attractiveness of an allocation unit, whereas its attractiveness is represented by the allocation weight *w*. For this reason, we call this rule an "attractiveness allocation heuristic". The allocation weight incorporates information on the profit improvement potential that results from assigning a higher budget to the allocation unit. This information includes the long-term marketing effectiveness of a product's marketing activity, the product's profit contribution level, and its growth potential.

FAWF (2011) suggest approximating the growth potential ρ by a multiplier that divides expected product revenues in 5 years (planning horizon) by its current revenue level. In our study, we follow FAWF (2011) by computing the expected product revenues based on the parameters of the growth function, which we specify subsequently. Note that the percentage-of-sales rule is a special case of Equation (2.1) if long-term marketing effectiveness, contribution margins, and growth potential multipliers are equal for all products. For application of the attractiveness allocation heuristic, r, cm, and RV are usually readily available from internal firm records, while δ, ε, and ρ must be estimated, e.g., by specifying an econometric model (FAWF 2011).

Numerical optimization. We employ a numerical optimization routine to obtain a unique solution to the dynamic optimization problem stated in equations (1.1)-(1.4). For this procedure, we need to specify the demand function q(ET+t,S,Z) and provide parameter values for r, p, c, etc. We then solve the optimization problem by applying the enhanced Generalized Reduced Gradient (GRG) 2 algorithm implemented in the Premium Solver Platform of Frontline Systems (for details, see Lasdon et al. 1978). The nonlinear optimization algorithm GRG2 iteratively varies the marketing allocation to maximize total discounted profits. It stops if the relative change in the objective is less than the convergence tolerance for the last five iterations. We set the convergence tolerance to the value of 10^{-10}. The constraints of our maximization problem are classified as active when they are within the range of 10^{-12} of one of their bounds.

Since we do not have a closed form solution, we numerically compute the Nash equilibrium in our competition scenarios by iteratively optimizing the budget allocation of one firm while holding the allocation of the competitor constant. When we apply this method consecutively for both competitors, we reach the Nash equilibrium if none of the competitors can improve its solution (Harcker 1984).

The big advantage of numerical optimization is that it generates optimal budgets for the specified problem. A disadvantage in practical application is, however, that the user must correctly specify the demand function and know the parameter values. While numerical optimization is always superior to heuristics under full information, it is interesting to compare the performance of the methods under more realistic conditions when demand parameters (e.g., sales elasticities) are subject to estimation error. An adaptive dynamic allocation approach that updates information on demand parameters might be able to deal with the uncertainty. The development of such an algorithm for the dynamic budget allocation under Nash competition is a contribution in its own and beyond the scope of this paper. It would also be unfair to compare the attractiveness heuristic (2.1)-(2.2) with an adaptive numerical optimization approach as it was derived under the assumption that the true demand parameters are known.

Experimental Design

Setup of the decision problem

We assume a firm using two types of marketing activities to promote sales of a product portfolio with four products. Simulation runs with larger portfolios and more marketing activities did not reveal significant differences compared to the results obtained from this firm setting but exponentially increase computation time. We assume the firm sets the total marketing budget at the end of each year. The task is then to find the optimal allocation of this budget across the four products and two activities, i.e. in total an allocation decision for eight allocation units has to be made. The discounted profit over the next five years is the objective criterion (see Equations 1.1-1.4 again). The budget planning process recurs every year. The firm may revise allocation decisions based on new market information that results from competitor moves, as an example.

We note that FAWF (2011) also report on a small simulation study. The objective of their study is to support the convergence of the proposed heuristic. As a result, they focus on just this heuristic and consider only 16 different market conditions. Our objective is to compare the performance of 4 allocation methods including numerical optimization with noisy demand parameters. The set of experimental factors is much broader leading to 1,024 different market conditions.

Data generation without estimation error

We design a Monte Carlo experiment, in which we experimentally manipulate 9 factors that can be divided into the following 5 groups:

1. Market response model: multiplicative model or modified exponential model;
2. Growth model: symmetric or asymmetric growth function;
3. Product characteristics: homogenous or heterogeneous values for 5 characteristics, namely sales elasticities, marketing carryover coefficients, sales levels, growth parameters, and product ages;
4. Competitive situation: no competition or Nash competition; and
5. Initial budget allocation: equal or proportional-to-sales initial allocation across products

We add a 10th factor subsequently when we introduce noisiness into demand parameters. Group 1, 2, 4, and 5 each includes one factor with two levels. Group 3 includes 5 factors, each with two levels. The full factorial design produces $2^9 = 512$ experimental conditions under which we use the heuristic rules and numerical optimization to generate allocation decisions. Adding parameter noisiness later increases the number to 1,024 conditions. Recall that the objective is to maximize discounted profit over the next five years. Consistent with our setup of the decision problem, we generate allocation decisions and the resulting discounted profit for 10 consecutive planning periods. Hence, we observe $512 \times 10 = 5{,}120$ allocations and their associated discounted profits, which we compare with the optimal solution. The observation of the performance of the decision rule over 10 planning periods enables us to investigate the convergence properties of the rule.

Factor 5 has no relevance when using the naïve and the percentage-of-sales rules. As a result, the number of total allocation decisions reduces to $2^8 = 256$ and $256 \times 10 = 2{,}560$, respectively, in these cases. Since we assume that the true values of all parameters are known, the numerical optimization method by definition yields the true optimum.

Market response model. Hanssens, Parsons, and Schultz (2001) discuss a variety of response functions that have been used in market research. The shape of the response function may affect the optimum and thus a rule's performance (Albers 2012). Note that models, which assume linear or increasing returns to scale, are not eligible because the optimal budget for an allocation unit would be zero or equal to the total budget. We therefore choose functional types that experience diminishing returns for higher levels of spending. Specifically, we use the multiplicative model and a modified exponential model. To keep notation low, we do not use indices for the factor levels. Unit sales q for product i in equation (1.1) are specified for the multiplicative model as follows:

$$q_{it} = a_i \cdot S_{1it}^{b_{1i}} \cdot S_{2it}^{b_{2i}} \cdot S_{cit}^{b_{ci}} \cdot g\left(ET_i + t, S\right), \tag{3.1}$$

where a_i is a scaling constant, and b_{1i} and b_{2i} are sales response parameters that determine marketing responsiveness with respect to own marketing stock variables S_1 and S_2. S_C and b_{Ci} measure the competitive total marketing stock and its cross-effect, respectively. $g(\cdot)$ represents the growth function, which we discuss subsequently. Sales elasticities, which we need as input for the attractiveness heuristic, are equal to the power coefficients. Note that they already measure the long-term impact of marketing expenditures. To obtain short-term effects, we need to multiply them with ζ_{in}, the decay coefficient from the difference equation (1.3). We use this equation to compute the marketing stock S. The scaling constant a_i is set in a way that it corresponds to the assumed initial sales levels, given all other variables and parameters for (3.1).

The multiplicative response model is by far the most frequent aggregate response model found in empirical research (Hanssens, Parsons, and Schultz 2001). It shows diminishing returns for response parameters between 0 and 1 and accommodates interaction effects among marketing activities. However, this specification has its limitations. It assumes constant elasticities and does not accommodate a saturation level for sales.

The modified exponential model allows for these effects and has seen several empirical applications to marketing spending models (Hanssens, Parsons, and Schultz 2001):

$$q_{it}=M_i\left[1\text{-}\exp\left(b_{1i}\sqrt{S_{1it}}+b_{2i}\sqrt{S_{2it}}+b_{ci}\sqrt{S_{cit}}\right)\right] g\left(ET_i+t, S\right), \tag{3.2}$$

where M_i is the market potential for product i and all other terms are defined as earlier. The square root of the marketing stock avoids allocation solutions where the budget is fully invested in only one of the two marketing activities, which are considered unrealistic by managers. Market potentials and response parameters are chosen in a way that they correspond to the assumed initial sales elasticities and product sales levels.

Growth model. The growth model describes the life cycle of a product. Fischer, Leeflang, and Verhoef (2010) show that marketing investments have the power to significantly shape the life cycle, i.e., the growth potential of a new product. The authors derive several parametric growth functions from a generalized growth model and differentiate between symmetric and asymmetric life cycles. We adopt their specifications for a symmetric life cycle and an asymmetric life cycle. Both specifications are highly flexible and allow capturing a multitude of different shapes and thus represent most forms of growth patterns observed in empirical studies:

$$g\left[ET_i+t, s\right]=\lambda(S)\cdot(ET_i+t)+\eta(S)\cdot(ET_i+t)^2, \qquad \text{(symmetric model)} \tag{4.1}$$

$$g\left[ET_i+t,s\right]=(ET_i+t)^{\lambda(S)}\cdot\exp\left[\eta(S)\cdot(ET_i+t)^2\right], \qquad \text{(asymmetric model)} \tag{4.2}$$

where λ and μ are the growth parameters which determine the shape of the life cycle in terms of their time-to-peak sales as well as their height-to-peak sales. Following Fischer et al. (2011), the growth parameters λ and η are influenced by the marketing stock according to

$\lambda(S) = \lambda_0 + .005 \cdot \ln(S)$, and $\eta(S) = \eta_0 + .00005 \cdot \ln(S)$. The parameters λ_0 and η_0 determine the life cycle when virtually no marketing investment is made. We set their values so that the time-to-peak sales and the height-to-peak sales are the same irrespective of the type of growth model.

Product characteristics. There are five product characteristics for which we create a situation of homogenous or heterogeneous parameter values across products (see Table 1; Tables follow References throughout.). Considering the profit maximization problem of (1.1)-(1.4), one could also think of varying the discount rate and the profit contribution margin. We did not do that because it does not generate new insights but only increases computational burden. Changes in the discount rate affect all products in equal measure. So, optimal allocations do not change much unless the rate is unrealistically high. Because profit margins just scale sales downwards or upwards, their variation does not add explication beyond varying the revenue level, which we do. We now discuss the experimental factors in detail.

Sales elasticity. We assume two marketing activities for each product that could be sales force and advertising, for example. Motivated by meta-analyses we choose an average elasticity of about .31 for sales force (Albers, Mantrala, and Sridhar 2010) and of .15 for advertising (Sethuraman, Tellis, and Briesch 2011). To reduce computational burden we vary only the sales force elasticity while keeping the advertising elasticity constant as this satisfies heterogeneity across marketing responsiveness.

Carryover coefficient. We set the average carryover coefficient to .5, which is the generalized value found in meta-analyses (e.g., Sethuraman, Tellis and Briesch 2011), and vary this value between .4 and .6. Larger carryovers are unrealistic for annual data and often do not give a unique solution. Smaller values are less interesting because they take out the dynamics, which we want to analyze.

Sales level. The sales level defines how much sales of the focal product are generated in the starting period. On average, we assume a sales level of 2.5m that varies between 1m and 4m.

Growth parameter. Following Fischer, Leeflang and Verhoef (2010), we assume that products reach their peak on average after 11 years and end their life cycle after approximately 25 years. In the asymmetric growth model, the parameter set λ_0= 1.1 and η_0= -.1 satisfies this assumption. In the symmetric model, the parameter η_0 needs to be adapted to -.05. Again, to reduce computational burden, we only vary one growth parameter, λ_0.

Product age. Generally, we assume that products are 3 years old in the initial period. We vary ages from 1 to 4 years. After 10 planning cycles, the oldest product will be in the market for 20 years.

Competitive situation. Assuming Nash competition, we simulate the dynamic game for two firms with a portfolio of 4 products and 2 marketing activities. Both firms face the same profit maximization problem (1.1)-(1.4). Each product has a direct competitive product in the portfolio of the other firm. One firm is exposed to all possible combinations of experimental factors. We randomly assign experimental conditions to the competitor firm. Trying all possible combinations across the two competitors yields up to 65,536 experimental conditions and 655,360 profit simulations depending on the rule, which increases computation time extensively without generating substantial new insights.

Our sales response functions (3.1) and (3.2) incorporate the influence of competitive marketing via the total marketing stock S_{Ci} of the competitive product of product i and the cross-effect b_{Ci}. We set the competitive marketing stock elasticity ε_C to -.10 across all products (e.g., Chintagunta and Desiraju 2005).

Initial budget allocation. We need to define the allocation of the total budget at the beginning of the very first decision cycle to initialize the marketing stocks for products and activities. Here, we use either the naïve rule or the percentage-of-sales rule to generate the initial budget allocation prior to the start of the simulation experiment. Dividing these budgets by the product-specific decay coefficient produces the steady-state stock levels associated with the initial budget allocation.

In the simulation, we change the initial budget allocation only for the attractiveness heuristic and the numerical optimization. For the naïve and the percentage-of-sales allocation rules, the initial allocation corresponds to the rule that is assumed in the decision process.

Data generation with estimation error

The assumption that managers know the true values of unobservable demand parameters is probably very unrealistic. For that reason, we impose an estimation error on demand parameters and generate data under all 512 experimental conditions again. Specifically, we impose an error on the response parameters for the two marketing activities b_1 and b_2 and on the growth parameters λ_0 and η_0. We do not consider an error for the carryover coefficient because that coefficient just scales short-term responsiveness to long-term responsiveness adding no additional insight but increasing computational complexity.

The simulation error is randomly generated for each parameter by drawing a number from a symmetric triangular distribution with the lower limit of -25 % of the parameter value and an upper limit of +25 % of the parameter value. A range of 25% is even larger than the standard deviation for generalized effects found in meta-analyses (e.g., Albers, Mantrala, and Sridhar 2010; Sethuraman, Tellis, and Briesch 2011). Specifically, the estimated parameters are obtained by:

$$\mu_{EP} = \mu_{TP} + \xi, \quad \xi \sim T\,(-0.25_{\mu TP}, 0.25_{\mu TP}) \tag{5}$$

where μ_{EP} is the estimated parameter value, μ_{TP} the true parameter value, and ξ is the error term. We use the triangular distribution to avoid that implausible values (e.g., negative response parameters) are generated that may happen with extreme value distributions.

To reduce overall computation time we use the technique of common random numbers, which is widely used in the simulation literature (Kleijnen and Groenendaal 1992). It requires taking the same set of random numbers for all simulation runs within a replication. This guarantees that all variations in the simulation outcome are only due to desired changes in the experimental variables and not due to random changes in the simulation environment. The random numbers only vary across replications. Consistent with previous simulation studies (e.g., Andrews, Ainslie and Currim 2008), we generate a total of three replications. This low number is sufficient because the purpose is not to simulate profit outcomes from realizations of a random variable. We rather want to understand the effect of an erroneous input variable, whereas the error should not be imposed arbitrarily but randomly chosen from a distribution.

Measure of performance

The key single objective is profit maximization. Thus, our performance measure (suboptimality) is defined by the extent to which discounted profits under a specific allocation rule differ from the profit generated with the true optimal allocation:

$$\text{Dev\_}\pi_t = \left(\pi_t^{\text{optimal}} - \pi_t^{\text{rule}}\right) / \pi_t^{\text{optimal}} \tag{6}$$

where $\Pi^{optimal}$ is the discounted profit generated with the optimal budget allocation and Π^{rule} is the discounted profit that results from budget allocation according to a specific rule.

Our performance measure is indexed by t because we simulate an annually recurring budget planning process. For the first planning cycle, discounted profit is obtained from years 1-5, for the second cycle from years 2-6, etc. We later also investigate the speed of convergence towards the optimal solution as a further performance characteristic.

Results

Performance of rules for demand parameters without error

The following results discussion addresses our first three research questions. We first report on each rule's degree of suboptimality. We then evaluate their convergence toward the optimal solution and finally discuss the rules' performance under extreme conditions (reliability of rule).

Degree of suboptimality. We assess the degree of suboptimality of an allocation rule by its average deviation from optimal profit across all market, firm, and competitive conditions. We use the term *optimal profit* to characterize the profit generated with the true optimal allocation. Assuming that the true demand parameters are known, numerical optimization produces the optimal solution. The profits generated with the other three methods deviate from the optimal profit. Table 2 shows the overall means and medians by rule and type of competition.

Under monopoly, the naïve rule is rather far away from generating optimal profits with an average deviation of 20.7%. The percentage-of-sales rule comes closer but still shows an average deviation of 10.3%. The best result is generated under the attractiveness heuristic that comes very close to the optimal profit by an average deviation of just .6%. Results are similar under

Nash competition. The attractiveness rule is still very close to the optimal profit with only .7% average profit deviation. Profits under the percentage-of-sales rule deviate on average 8.7% from optimal profit. The deviation reaches 22.0% under the naïve rule.

Convergence of rules. Table 2 shows the average deviations from optimal profit across experimental conditions for each planning cycle. Results for the naïve rule suggest that performance deteriorates over time. While the mean profit deviation is 18.5% under monopoly (19.4% under Nash competition) in the first decision period, it increases to 22.2% (23.7%) in the last period. In contrast, results for both the percentage-of-sales rule and the attractiveness heuristic improve over time (see table 2 for details). Based on the simulation results alone, we cannot say whether the methods converge to the true optimum or some other boundary value. According to Fischer et al. (2011), the true optimum is likely to be the convergence limit for the attractiveness heuristic, which is a contraction mapping method that replaces an allocation subsequently by allocations closer to the true optimum. Indeed, mean deviations from optimal profit reduce at a much higher annualized rate of -21.2% under monopoly (-22.1% under Nash competition) for the attractiveness heuristic compared to a rate of -1.4% (-3.1%) for the percentage-of-sales rule.

Reliability of rules. The managerial value of average performance numbers is limited since it is not clear how the decision rule performs under extreme conditions. We consider a decision rule as reliable if the variance of profit deviations across market conditions and the maximal deviation under worst conditions are small. Table 2 presents the overall standard deviation, the overall maximum, and the maximum in the last decision period for our key performance measure.

The results suggest that the naïve rule is not only largely suboptimal but also highly unreliable. Standard deviation in profit deviations amounts to 10.2% under monopoly (9.3% under Nash competition). The maximal deviation overall and in the last decision period is as high as 44.4% (40.0%). Results are somewhat better for the percentage-of-sales rule but not small (see table 2 for details). Standard deviation and maximal deviation from optimal profit are lowest for the attractiveness heuristic. The attractiveness heuristic appears to be quite reliable. Its standard deviation of the performance measure amounts to only .8% under monopoly (.9% under Nash competition). The maximal deviation from the profit optimum across 5,120 decisions is only 5.7%. Due to its convergence property, this distance even reduces to only 1.2% in the last decision period.

Performance of rules for demand parameters with error

We now discuss results for conditions when true parameters for marketing responsiveness and product growth are unobservable and subject to error. Hence, we address the fourth research question that asks for the impact of estimation error on the performance of rules. Since only numerical optimization and the attractiveness rule make use of this information the discussion is limited to these two methods.

Table 3 reveals a remarkable result that may appear counterintuitive at first glance. Across all evaluation criteria, we note that numerical optimization performs worse than the attractiveness heuristic. Numerical optimization results are no longer optimal but, on average, deviate from the optimal profit by 2.9% under monopoly. Results do not improve from the first to the last decision period but get worse (see table 3 for details). The method also does not appear to be reliable with a high maximal deviation from optimal profit of 24.6%. The statistics are even worse under competition.

The performance statistics for the attractiveness heuristic also decline compared to the situation without noisy demand parameters. However, the decline is modest. Profits in the final decision period deviate from optimal profit, on average, only by .4% under monopoly (.5% under Nash competition). This is an improvement of more than 85% over the results obtained with numerical optimization.

How can we explain this sharp difference in the performance of the two methods? Both methods process demand parameters that are erroneous. None of the two methods was designed as an adaptive method with the purpose to actively handle the uncertainty in demand parameters, e.g., by incorporating some form of updating process. As a result, both methods still process the erroneous information in every new decision period again. It appears, however, that the attractiveness heuristic is able to incorporate feedback from the market in terms of actually realized product sales of the previous decision period. Note that this information goes directly into the allocation weight (see again Equation 2.2). Even though the elasticity and growth multiplier estimates are still not the true ones, previous period's sales carry information about their true values. In contrast, the numerical optimization algorithm cannot use such feedback but needs to fully rely on the noisy parameters. The error seems to propagate across subsequent planning periods. Hence, the attractiveness heuristic's disadvantage of not using implicit optimal sales levels, which are part of the optimality condition, but actual past sales turns into an advantage if demand parameters are noisy.

Influence of Experimental Conditions on the Performance of Rules

Analysis of main effects

In this section, we address the last of our 5 initial research questions. Specifically, we want to understand which experimental factors drive the suboptimality of allocation methods most, whether this affects all methods similarly, and how the influence of a factor develops over time.

Table 4 presents the mean deviations from optimal profit by rule and experimental condition. Significance levels for mean differences between factor levels are denoted with an asterisk. A large mean difference between levels suggests that the associated factor has a greater influence on performance compared to other factors. With this interpretation in mind, it is apparent from the table that there is no single market, firm, or competitive factor that stands out from all other. However, we find that product heterogeneity in terms of marketing responsiveness and product age are among the most important drivers of performance across methods. Heterogeneity of sales elasticities leads to significantly larger deviations from optimal profit than homogenous elasticities for the naïve rule (Δ_{mean} = +11.9%), the percentage-of-sales rule (Δ_{mean} = +8.7%), and the attractiveness heuristic (Δ_{mean} = +.2%). If product ages are heterogeneous in the portfolio the mean deviation from optimal profit is especially large for numerical optimization (Δ_{mean} = 13.9%) and the naïve allocation rule (Δ_{mean} = 10.5%) and relatively large for the percentage-of-sales rule (Δ_{mean} = 1.4%) and the attractiveness heuristic (Δ_{mean} = .2%). In addition, the existence of an estimation error appears to drive deviations from optimal profit for both numerical optimization (Δ_{mean} = 3.0%) and the attractiveness heuristic (Δ_{mean} = .3%). Recall that this factor does not apply to the other two methods.

Interestingly, we find that competition, initial conditions such as the heterogeneity in product sales levels and the initial budget allocation, as well as the type of the growth model and the response model are less relevant for driving the performance of the allocation methods. Mean differences are generally small, sometimes even insignificant ($p > .05$). The exception is that the heterogeneity of product sales levels largely impacts the performance of the naïve allocation ($\Delta_{mean} = 7.8\%$). Similarly, the type of market response model makes a difference for numerical optimization ($\Delta_{mean} = 4.0\%$) and the percentage-of-sales rule ($\Delta_{mean} = 2.0\%$).

Analysis of interactions with time

Specification of regression model. An important question is whether our conclusions from the univariate test of mean differences between factor levels holds in the multivariate case, which considers the joint role of all experimental factors together. In addition, we are interested in the interaction of experimental factors with time, i.e., whether their impact on rule performance changes over time. For this purpose, we specify the following regression model and estimate the model for each method:

$$\begin{aligned} \text{Dev\_}\pi_{dlz} = \; & \alpha + \beta_1 \cdot \text{Fac\_Elast}_l + \beta_2 \cdot \text{Fac\_Sale}_l + \beta_3 \cdot \text{Fac\_Grow}_l + \beta_4 \cdot \text{Fac\_Car}_l \\ & + \beta_5 \cdot \text{Fac\_ET}_l + \beta_6 \cdot \text{Fac\_GrM}_l + \beta_7 \cdot \text{Fac\_RespM}_l + \beta_8 \cdot \text{Fac\_lniB}_l \\ & + \beta_9 \cdot \text{Fac\_Comp}_l + \beta_{10} \cdot \text{Fac\_Err}_l + \beta_{11} \cdot z + \gamma \cdot \left[z \cdot \Gamma_l \right] + e_{dlz}, \end{aligned} \tag{7}$$

where

$\text{Dev\_}\Pi_{dlz}$	: Deviation from optimal profit for scenario l and replication d in planning cycle z;
Fac_Elast	: Heterogeneity of elasticities across products (0: equal, 1: unequal);
Fac_Sale	: Heterogeneity of sales level across products (0: equal, 1: unequal);
Fac_Grow	: Heterogeneity of growth parameters across products (0: equal, 1: unequal);
Fac_Car	: Heterogeneity of carryover coefficients across products (0: equal, 1: unequal);
Fac_ET	: Heterogeneity of elapsed time since launch across products (0: equal, 1: unequal);
Fac_GrM	: Type of growth model (0: asymmetric, 1: symmetric);
Fac_RespM	: Type of market response model (0: multiplicative, 1: mod. exponential);

Fac_IniB	: Initial budget allocation (0: equal allocation, 1: proportional to sales);
Fac_Comp	: Competitive situation (0: No competition, 1: Nash competition);
Fac_Err	: Estimation error in demand parameters (0: non-included, 1: included);
Γ_l	: Vector of experimental factors in scenario l;
α, β, γ	: (Unobserved) parameters;
e	: Error term;
z	= 1, 2, …, 10 (index for planning cycles or periods, respectively);
l	= 1, 2, …, 1024 (index for scenarios/experimental conditions); and
d	= 1, …, D_l (index for replications).

All scenarios of our simulation experiment that do not include erroneous demand parameters are independent, i.e. $e_{dlz} \sim N(0, \sigma^2)$, with error variance σ^2. As a result, OLS estimation is efficient for the naïve allocation method and the percentage-of-sales rule. For the attractiveness heuristic and numerical optimization, we impose an error term on demand parameters that induces a correlation among regression errors (Kleijnen 1988). The errors for each replication across scenarios as well as across planning cycles within a scenario are correlated, while the errors across replications within a scenario are uncorrelated, i.e. $e_{dlz} \sim N(0, \sigma^2_{lz})$, with variance σ^2_{lz} and $Cov(e_{lz}, e_{lz'}) = \sigma_{lz,lz'}$ for $lz \neq lz'$. We account for these correlations by specifying a generalized variance-covariance matrix (Greene 2006).

Results. Estimation results of Equation (7) are shown for each allocation method in Tables 5a and 5b. The first column by method includes estimated main effects whereas the second column includes estimated interaction effects with respect to time. We first note that the estimation results for main effects support our conclusions on the role of experimental factors that we draw from the preceding univariate analysis of mean differences (see table 4 again). We do not repeat them here but focus on the issue whether the impact of factors changes over time when the allocation method is repeatedly applied.

The overall impact of time on the performance of a method is given by the main-effect coefficient with respect to time. We find positive, significant effects ($p < .01$) with respect to the naïve allocation method and numerical optimization. This supports our earlier conclusions that the performance of these methods deteriorates with time. Consistent with our previous discussion of table 2 and 3, we find that the performance of the attractiveness rule improves over time. The associated coefficient is negative and significant ($p < .01$), which suggests that the deviation from optimal profit decreases over time irrespective of the influence of experimental factors. The coefficient is also negative for the percentage-of-sales rule, but not significant ($p > .05$).

This picture does not change when we consider the interaction of a factor with time.[3] The associated coefficient measures whether the influence of a factor on the deviation from optimal profit amplifies (same sign as the main effect) or diminishes (opposite sign of the main effect) over time. A non-significant coefficient signals that time has no impact on the relevance of a factor. For the naïve allocation, Table 5a reveals that time has no impact on the role of most factors and even amplifies the role of two factors. This adds to the overall negative impact of repeated application of the rule on its performance. In contrast, the importance for driving the performance of the percentage-of-sales rule diminishes over time for 5 out of 8 factors. In total, these findings seem to explain the slow but consistent convergence of the rule's performance to the optimal profit as shown in table 2 and 3. Table 5b shows that the performance impact of all factors significantly decreases over time for the attractiveness rule. This result underlines the impressive convergence property of the rule (see table 2 and 3 again). Finally, we find that 4 out of 9 coefficients for time interactions are not significant or show the same sign as the main effect for numerical optimization. Together with the positive main effect of time this suggests that the performance of the method rather deteriorates than improves over time.

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[3] Equation (7) assumes a linear convergence process. We also tested as a log-linear process, i.e. t was replaced by Log(t). Estimation results are very similar.

Conclusions and Future Research

In this experimental simulation study, we investigate the performance of 4 methods to allocate a fixed marketing budget across products and marketing activities in order to maximize discounted portfolio profit over a five-period planning horizon. Consistent with the idea of a recurring planning process, allocation decisions are revised each period by repeatedly applying the respective decision rule. Our study reveals important insights into the performance characteristics of the methods that can be summarized as follows:

1. Simple allocation rules such as an equal budget allocation and an allocation proportional to sales are, on average, rather far from being optimal. A contraction-mapping rule such as the attractiveness heuristic is remarkable close to the optimum.
2. If true demand parameters are not known but estimated with an error, numerical optimization no longer produces optimal results. In fact, its suboptimality is considerably higher than that of the attractiveness heuristic.
3. Under extreme conditions, only the attractiveness heuristic appears to be reliable. Deviations from optimal profit may be quite large with the other methods.
4. The attractiveness heuristic converges quickly to the optimal solution if repeatedly applied over time. The same is true for the percentage-of-sales rule, albeit at much slower speed. Numerical optimization (with noisy demand parameters) and naïve allocation deteriorate in their profit results over time.
5. Across methods, heterogeneity in marketing responsiveness and product age appear to influence the performance of methods most. Deviation from optimal profit is higher the larger the heterogeneity across the portfolio is.

We believe that these insights have important ramifications for both research and practice. The probably most surprising new result is that an exact method such as numerical optimization turns out to be inferior to a decision heuristic if it is applied under the realistic assumption that true demand parameters are not known. Given that we are not aware of a published optimization approach for the dynamic portfolio maximization problem under competition and uncertain demand parameters, we would not recommend managers to use existing models but rather adopt a superior heuristic. Not every heuristic, however, is appropriate. Those that are theoretically derived such as the attractiveness heuristic demonstrate a remarkable reliable performance.

Researchers might accept the challenge to develop an optimization approach for the addressed complex allocation problem that can handle uncertainty in demand parameters. It would tremendously add to the practicality of such solutions. They should also pay attention to derive an advanced heuristic from the newly stated optimization problem including uncertainty.

Considering the influence of market, firm, and competitive conditions, we conclude that those factors that are associated with the effectiveness of marketing seem to have the greatest influence on optimal allocation results. Our analysis of the role of experimental factors revealed that the heterogeneity of marketing responsiveness and product age are most critical for the performance of an allocation method. Since we assume in our problem setting that marketing investments do impact the life cycle of a product, heterogeneity in product ages poses a challenge to the allocation decision. Young products, as an example, require enough resources to exploit their future sales promises. It is because of that growth potential that marketing managers need to trade-off investing in young but loss-making products compared to older, profitable products. It is a challenge for the allocation method to incorporate these dynamic effects.

Our study is also subject to limitations. The simulation factors are limited to the factors that characterize our profit maximization problem setting. For other settings, other factors might be relevant. However, we believe that this setting is quite realistic as it considers various forms of dynamics, portfolio effects, several marketing activities, competition, and noisy demand parameter information. Our focus is on estimation error in demand parameters, but not on specification error, i.e. assuming a wrong demand model. Since only numerical optimization requires the explicit specification of a demand model but not the other methods, we believe that the performance of that method is negatively affected. This raises even more concern about the practical application value of numerical methods.

For future studies we recommend to analyze and compare the performance of new and old heuristic methods developed in marketing science by a dynamic comprehensive simulation framework, as developed in this study, which also impose an estimation error on demand parameters to simulate realistic scenarios. Hopefully our work will motivate such efforts.

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Table 1. Experimental variation of product characteristics

		Product A	Product B	Product C	Product D
Sales elasticities					
Marketing activity 1	Homogenous	.33	.32	.31	.30
	Heterogeneous	.50	.49	.12	.11
Marketing activity 2	Homogenous	.15	.15	.15	.15
	Heterogeneous		(.15) *no variation*		
Marketing carryover coefficient δ [1]					
	Homogenous	.50	.50	.50	.50
	Heterogeneous	.60	.40	.40	.60
(Initial) sales level					
	Homogenous	2.5m	2.5m	2.5m	2.5m
	Heterogeneous	3.0m	4.0m	2.0m	1.0m
Growth parameter (symmetric model)					
Parameter λ_0	Homogenous	1.1	1.1	1.1	1.1
	Heterogeneous	.95	1.0	1.1	1.2
Parameter η_0	Homogenous	-.05	-.05	-.05	-.05
	Heterogeneous		(-.05) *no variation*		
Growth parameter (asymmetric model)					
Parameter λ_0	Homogenous	1.1	1.1	1.1	1.1
	Heterogeneous	.95	1.0	1.1	1.2
Parameter η_0	Homogenous	-.10	-.10	-.10	-.10
	Heterogeneous		(-.10) *no variation*		
Product age (elapsed time in years)					
	Homogenous	3	3	3	3
	Heterogeneous	1	2	3	4

[1] We do not model different carryover coefficients across marketing activities.

Table 2. Deviation from optimal profit means by rule and type of competition assuming no error in demand parameters

	Naïve allocation		Percentage-of-sales rule		Attractiveness heuristic		Numerical Optimization[1)]	
	Monopoly	Nash	Monopoly	Nash	Monopoly	Nash	Monopoly	Nash
Planning cycle								
1st	.18501	.19378	.11252	.10620	.01950	.02008	-	-
2nd	.19564	.20927	.11136	.10123	.01417	.01562	-	-
3rd	.20134	.21414	.10762	.09442	.00937	.01048	-	-
4th	.20523	.21723	.10423	.08886	.00610	.00688	-	-
5th	.20837	.22057	.10175	.08486	.00409	.00456	-	-
6th	.21120	.22351	.10009	.08212	.00291	.00318	-	-
7th	.21398	.22653	.09906	.08028	.00227	.00225	-	-
8th	.21670	.22969	.09847	.07906	.00194	.00197	-	-
9h	.21949	.23306	.09817	.07828	.00181	.00174	-	-
10th	.22255	.23674	.09809	.07781	.00180	.00165	-	-
Overall mean	.20706	.22045	.10313	.08731	.00640	.00684	-	-
Overall median	.21171	.21614	.09831	.09635	.00360	.00346	-	-
Overall Std. Dev.	.10179	.09284	.05371	.04264	.00809	.00875	-	-
Overall Maximum	.44381	.40020	.21910	.18682	.05113	.05727	-	-
Maximum for 10th planning cycle	.44381	.40020	.19810	.13304	.01070	.01158	-	-

[1)] The deviation from maximum profit is per definition zero for the solution of the numerical optimization as it determines the optimal solution.

Table 3. Deviation from optimal profit means by rule and type of competition assuming error in demand parameters

	Naïve allocation		Percentage-of-sales rule		Attractiveness heuristic		Numerical Optimization	
	Monopoly	Nash	Monopoly	Nash	Monopoly	Nash	Monopoly	Nash
Planning cycle								
1st	.18501	.19378	.11252	.10620	.02116	.02960	.02612	.03118
2nd	.19564	.20927	.11136	.10123	.01561	.01887	.02744	.03085
3rd	.20134	.21414	.10762	.09442	.01094	.01308	.02785	.03086
4th	.20523	.21723	.10423	.08886	.00785	.00931	.02789	.03046
5th	.20837	.22057	.10175	.08486	.00596	.00707	.02870	.03069
6th	.21120	.22351	.10009	.08212	.00485	.00580	.02918	.03101
7th	.21398	.22653	.09906	.08028	.00421	.00513	.03041	.03138
8th	.21670	.22969	.09847	.07906	.00387	.00479	.03070	.03188
9h	.21949	.23306	.09817	.07828	.00371	.00467	.03169	.03263
10th	.22255	.23674	.09809	.07781	.00367	.00467	.03281	.03382
Overall mean	.20706	.22045	.10313	.08731	.00818	.01029	.02932	.03148
Overall median	.21171	.21614	.09831	.09635	.00634	.00627	.01582	.01363
Overall Std. Dev.	.10179	.09284	.05371	.04264	.00935	.00960	.04073	.03517
Overall Maximum	.44381	.40020	.21910	.18682	.06397	.07970	. 24645	.26181
Maximum for 10th planning cycle	.44381	.40020	.19810	.13304	.01257	.01756	.24645	.26181

Table 4. Deviation from optimal profit means by rule and experimental condition

	Factor	Naïve allocation	Percentage-of-sales rule	Attractiveness heuristic	Numerical optimization
Product characteristics	**Sales elasticities**				
	Homogenous	.15435**	.05164**	.00738**	.03193**
	Heterogeneous	.27405**	.13881**	.00979**	.02887**
	Sales level				
	Homogenous	.17485**	.09548^{ns}	.00872*	.03388**
	Heterogeneous	.25356**	.09496^{ns}	.00845*	.02691**
	Growth parameters				
	Homogenous	.20271**	.09336**	.00882**	.03254**
	Heterogeneous	.22570**	.09709**	.00835**	.02826**
	Carryover coefficient				
	Homogenous	.21356^{ns}	.09503^{ns}	.00634**	.02951**
	Heterogeneous	.21485^{ns}	.09541^{ns}	.01083**	.03129**
	Product age				
	Homogenous	.16163**	.08848**	.00753**	.04261**
	Heterogeneous	.26678**	.10197**	.00964**	.18185**
Growth model					
	Asymmetric	.21716**	.09572^{ns}	.00916**	.03067^{ns}
	Symmetric	.21125**	.09473^{ns}	.00800**	.03012^{ns}
Market response model					
	Multiplicative	.21764**	.08504**	.00869^{ns}	.05039**
	Modified exp.	.21076**	.10541**	.00848^{ns}	.01041**
Competitive situation					
	Monopoly	.20795**	.10314**	.00774**	.02932**
	Nash competition	.22045**	.08731**	.00943**	.03148**
Initial budget allocation					
	Equal distribution	-	-	.00913**	.02933**
	Proportional to sales	-	-	.00804**	.03147**
Error in demand parameters					
	Not included	-	-	.00662**	.0000[1)]
	Included	-	-	.00924**	.03040**
Overall mean		.21420	.09522	.00859	.03040

Notes: ** p<.01; * p<.05; ^{ns} = not significant (Difference between the two means, based on ANOVA F-test)
1) No deviation from optimal profit by definition.

Table 5a. Experimental factors influencing the deviation from maximum profit: regression coefficients (standard errors) I

		Naïve allocation				Percentage-of-sales rule			
		Main effects		Interaction with time		Main effects		Interaction with time	
Factor	Level	Est. coeff.	Est. std. dev.	Est. coeff.	Est. std. dev.	Est. coeff.	Est. std. dev.	Est. coeff.	Est. std. dev.
Constant		.041	(.005)**			.042	(.002)**		
Sales elasticities	Homogenous		0		0		0		0
	Heterogeneous	.118	(.003)**	3×10^{-4}	(.001)	.096	(.001)**	-.002	(2×10^{-4})**
Sales level	Homogenous		0		0		0		0
	Heterogeneous	.084	(.003)**	-.001	(.001)	-.005	(.001)**	.001	(2×10^{-4})**
Growth parameters	Homogenous		0		0		0		0
	Heterogeneous	.010	(.003)**	.002	(.001)**	.005	(.001)**	-3×10^{-4}	(2×10^{-4})
Carryover coefficient	Homogenous		0		0		0		0
	Heterogeneous	.008	(.003)**	-.001	(.001)**	.003	(.001)**	-.001	(2×10^{-4})**
Product age	Homogenous		0		0		0		0
	Heterogeneous	.081	(.003)**	.004	(.001)**	.032	(.001)**	-.003	(2×10^{-4})**
Growth model	Asymmetric		0		0		0		0
	Symmetric	-.009	(.003)**	.001	(.001)	-.003	(.001)**	4×10^{-4}	(2×10^{-4})*
Market response model	Multiplicative		0		0		0		0
	Modified exp.	4×10^{-4}	(.003)	-.001	(.001)**	.012	(.001)**	.001	(2×10^{-4})**
Competitive situation	Monopoly		0		0		0		0
	Nash competition	.011	(.003)**	3×10^{-4}	(.001)	-.008	(.001)**	-.001	(2×10^{-4})**
Initial budget allocation	Equal distribution								
	Proportional to sales								
Error in demand parameters	Not included								
Time (# planning cycle)		.002	(.001)**			-2×10^{-4}	(3×10^{-4})		
(Pseudo) R^2		.868				.915			
# of observations		2,560				2,560			

Notes: ** $p< .01$, * $p< .05$

The factors initial budget allocation and estimation error are not included in our analysis of the naïve solution and the percentage-of-sales-rule because there is no variation for these factors. Numerical optimization is based in simulations with error in demand parameters only.

Table 5b. Experimental factors influencing the deviation from maximum profit: regression coefficients (standard errors) II

		Attractiveness heuristic				Numerical Optimization			
		Main effects		Interaction with time		Main effects		Interaction with time	
Factor	Level	Est. coeff.	Est. std. dev.	Est. coeff.	Est. std. dev.	Est. coeff.	Est. std. dev.	Est. coeff.	Est. std. dev.
Constant		.001	(4×10^{-4})**			.004	(.002)**		
Sales elasticities	Homogenous		0		0		0		0
	Heterogeneous	.008	(2×10^{-4})**	-.001	$(.4\times10^{-4})$**	-.004	(4×10^{-4})**	3×10^{-4}	$(.6\times10^{-4})$**
Sales level	Homogenous		0		0		0		0
	Heterogeneous	-.001	(3×10^{-4})**	1×10^{-4}	$(.4\times10^{-4})$*	-.003	(.001)**	-1×10^{-4}	$(.8\times10^{-4})$*
Growth parameters	Homogenous		0		0		0		0
	Heterogeneous	-.001	(2×10^{-4})**	1×10^{-4}	$(.4\times10^{-4})$*	-.003	(.001)**	-5×10^{-5}	$(.8\times10^{-4})$
Carryover coefficient	Homogenous		0		0		0		0
	Heterogeneous	.009	(3×10^{-4})**	-.001	$(.4\times10^{-4})$**	.003	(.001)**	-.001	$(.7\times10^{-4})$**
Product age	Homogenous		0		0		0		0
	Heterogeneous	.007	(2×10^{-4})**	-.001	$(.3\times10^{-4})$**	.015	(.001)**	-.001	$(.5\times10^{-4})$**
Growth model	Asymmetric		0		0		0		0
	Symmetric	-.002	(2×10^{-4})**	3×10^{-4}	$(.3\times10^{-4})$**	-.001	(.001)	3×10^{-4}	$(.5\times10^{-4})$**
Market response model	Multiplicative		0		0		0		0
	Modified exp.	3×10^{-4}	(2×10^{-4})	-1×10^{-4}	$(.3\times10^{-4})$**	-.031	(.001)**	6×10^{-5}	$(.5\times10^{-4})$
Competitive situation	Monopoly		0		0		0		0
	Nash competition	.001	(2×10^{-4})**	-2×10^{-4}	$(.3\times10^{-4})$**	.005	(.002)**	9×10^{-5}	$(.8\times10^{-4})$
Initial budget allocation	Equal distribution		0		0		0		0
	Proportional to sales	-.007	(2×10^{-4})**	.001	$(.3\times10^{-4})$**	.005	(.001)**	-.001	$(.5\times10^{-4})$**
Error in demand parameters	Not included		0		0				
	Included	.003	(2×10^{-4})**	-1×10^{-4}	$(.2\times10^{-4})$**				
Time (# planning cycle)	Planning cycle	-.001	(1×10^{-4})**			.001	(2×10^{-4})**		
(Pseudo) R^2		.820				.691			
# of observations		20,480				15,360			

Notes: ** $p< .01$, * $p< .05$

The factor estimation error is not included in our analysis of the numerical optimization method because there is no variation for this factor.