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# MARKET POSITIONING USING CROSS-REWARD EFFECTS IN A COALITION LOYALTY PROGRAM

Valeria Stourm, Eric T. Bradlow, Peter S. Fader<sup>1</sup>

## ABSTRACT

While single-brand reward programs encourage customers to remain loyal to that one brand, coalition programs encourage customers to be “promiscuous” by offering points redeemable across partner stores. Despite the benefits of this “open relationship” with customers, store managers face uncertainty as to how rewards offered by partners influence transactions at their own stores. We use a model of multi-store purchase incidence to show how the value of points shared among partner stores can explain patterns in customer-level purchases across them. The model is used to empirically test hypotheses on how reward spillovers among partners are driven by: (1) differences in policies on reward redemption, (2) the overlap in product categories between stores, and (3) geographic distance between stores within a city. In addition, we leverage variation generated by a natural experiment, i.e., a devaluation of the program’s points, to demonstrate how the value of points influences the positioning of partner stores within the coalition and the purchasing patterns across them. We conclude by delineating some managerial implications for the design of a coalition’s reward policies, including a simulation showing that customer-centric targeted rewards can be an effective strategy to compensate for the devaluation.

Keywords: loyalty programs, rewards, retailing, Bayesian estimation

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## Motivation

A typical single-brand loyalty program encourages repeat purchases by offering points redeemable for future discounts. While these programs have proliferated, many retailers have found them to be ineffective (Hlavnicka and Sullivan 2011): if customers tend to visit infrequently, they may earn rewards too slowly to care about them, and thus the program may fail to motivate increased customer behavior. Such stores may instead benefit from participating in a *coalition loyalty program*: a scheme to provide rewards that can be earned and redeemed faster and with greater flexibility across stores (Breugelmans et al., 2015, Danaher, Sajtos, and Danaher 2017).

While the goal of single-brand reward programs is to foster loyalty by giving points that encourage customers to remain loyal to that one brand, coalition programs instead encourage customers to be “promiscuous” by offering points that are redeemable at other partner stores. For example, a shoe retailer may reward customers with points that can be redeemed not only at its stores, but also at a clothing store that is a partner of the same coalition program. The more a store’s customers are likely to purchase at other partner stores, the faster they will earn points and the more they will value the reward currency.

Despite the benefits of this type of “open relationship” with customers, store managers face uncertainty as to how rewards earned at other partners influence their own transactions. Managers are left wondering: how does the value of the points shared across the coalition’s partners (i.e., the value of the reward currency) influence how customers purchase across partner stores? Are stores influenced more by the rewards offered by partners competing in the same product categories, or by partners that geographically nearby? Are more generous store-level reward policies (for example, offering higher reward rates or accepting redemptions) effective levers to compete

with other partners?

These important managerial questions, absent from single-brand programs, are challenging because stores within a coalition often do not directly compete in the same product categories. Interestingly, while successful coalition programs in Europe tend to include hundreds of competing retailers (i.e., Nectar in the UK), emerging coalition programs in the United States instead rarely include any partners that sell the same product categories. One recent example is Plenti, an American coalition program launched in 2015 with partners such as AT&T (which withdrew in October 2017), Exxon Mobil, Macy's, Nationwide Insurance, Rite Aid Pharmacy, and Hulu (Tierney 2017). While these research questions are specific to coalition programs, they may become increasingly pertinent to single-brand program managers as several innovations further increase the fungibility of rewards across programs. For example, some brands such as Best Western Hotels & Resorts match rewards to high-value customers who have earned high levels of rewards at competitors.

This paper uses a unique dataset obtained from a European coalition loyalty program (via the Wharton Customer Analytics Initiative, [wcai.wharton.upenn.edu](http://wcai.wharton.upenn.edu)) to empirically show how the value of a shared reward currency influences how partner stores compete with, or complement each other, through cross-reward effects. We model the purchase incidence across forty stores within a European city. Some stores compete directly by selling goods within the same product categories, while others do not. Store policies on how customers earn and redeem points also differ: the rate at which customers earn points differs across stores, and points can only be redeemed at select partners that choose to allow it. We are able to measure cross-reward effects from exogenous variation in reward rates at different points in time and across different stores, including a one-time “shock” in 2009: program managers (not individual stores) devalued the coalition's points to a third of their original value, while at the

same time making it easier for customers to redeem rewards. Although these policy changes occurred at the same time, we can discern the effects of each because some partner stores accept redemptions while others do not.

We develop a Network Affinity Model (NAM) to measure cross-reward effects while accommodating the diversity of store types. The model views the coalition loyalty program as a network in which stores are nodes and latent links between them represent cross-reward effects sent and received across them. Each link measures how much the purchase propensity for one store changes with an increase in the value of rewards offered by other partners. The links can capture both positive and negative associations in customer-level purchase incidence across multiple stores, and may be asymmetric. The parameters that govern these links are used to test hypotheses of how rewards at one partner store affect purchases at others. The hypotheses relate cross-reward effects to: (1) store policies on reward redemptions, (2) how similar stores are in terms of category and geographic affinity, and (3) the devaluation of points.

We construct “heat maps” from the links to visualize how partner stores compete or complement each other through the reward rates offered to their customers. While conventional store choice models typically compare competitors within an industry, the links in our model characterize the market positioning “landscape” of coalition partners that operate in different retail categories. By positioning partner stores with a common reward spillover metric (i.e., the links), even though several do not sell the same product categories, we contribute to a call for research that can “detect relationships among brands (in our case, stores) that lie outside the conventional definition of product category” (Elrod et al. 2002, p. 230). We also quantify the asymmetry between links received versus sent with measures of competitive clout and vulnerability (Kamakura and Russell 1989) that summarize the degree to which each

partner influences, and is influenced by, the reward rates of other coalition partners.

These store-level metrics are used to illustrate through a counterfactual simulation how a customer-centric targeting strategy can help to counteract the negative consequences of the points devaluation. We simulate a counterfactual scenario in which a gold tier is created that includes the five “best” customers from each partner store. This select group earns points three times as fast, effectively earning at the pre-devaluation reward rates. We find that adding a customer-centric component to the coalition program can be an effective strategy to restore the ability of stores to differentiate among themselves through policy differences in how customers earn and redeem points across them.

In summary, this paper conducts analyses showing how a coalition’s reward currency influences how customers purchase across partner stores. These include: (1) measuring reward cross effects, (2) testing hypotheses on how these effects vary across stores, (3) visualizing the market structure of the coalition, (4) calculating metrics that summarize how much stores influence and are influenced by others’ rewards, and (5) analyzing how cross-reward effects change when the value of the shared reward currency is high versus low. These analyses can help coalition managers advise partner stores on their specific reward policies, identify partners likely to mutually benefit from cross-marketing opportunities, and assess the design of the reward policies across the coalition. Our findings are also relevant to firms providing rewards across an umbrella of services, such as theme parks and casinos, that might want to understand cross-category complementarity or substitution.

Our work adds empirical evidence in a nascent research area that has previously failed to find strong spillover effects across retailers. Sharp and Sharp (1997) found that only two out of six brands in a coalition program in Australia seemed to positively benefit from the partnership. Similarly, Dorotic et al. (2011) failed to find spillover

effects of store-specific point promotions for the top five non-competing retailers of a Norwegian coalition program. Nevertheless, cross-effects have been found in a related setting (Pancras, Venkatesan and Li, 2015) for 26 competing restaurants, each operating their own independent loyalty program (in which rewards were not shared across partners). In their setting, a focal restaurant in a city is affected by negative cross effects of rewards from nearby restaurants but positive cross effects from restaurants further away. More recently, Danaher, Sajtos, and Danaher (2017) study how consumers redeem from an online catalog with points earned through a coalition, but do not consider how points are earned across stores.

The rest of the article is organized as follows. We first describe the data, including a timeline of the reward policy changes to the coalition program. Then we develop the NAM framework for multi-store purchase incidence that can test hypotheses of how changes in rewards may have affected how customers purchase across partner stores. Afterwards, we use the model estimates to evaluate model fit relative to benchmarks, to test hypotheses on cross-reward effects, to position the stores of the coalition in a latent space, and to assess the impact of the reward currency devaluation and other changes in the program redemption policies. Finally, we provide a discussion of future research directions.

## Data Description

Our data includes credit card transactions across 76 stores in a European city that is internationally known as one of the major retail centers of the world. Stores are members of a coalition program with presence throughout the country. This paper focuses on how customers purchase across the 40 partner stores that had active transactions between January 2006 and December 2012, the observation period of our dataset. Purchases at the remaining 36 stores with sparser data are used to identify purchase

occasions in which a customer purchased in the city but not at the focal stores. Across 1,636 customers who purchased at least once across these focal 40 stores, we observe 16,984 transactions of which 12,304 were made at the focal 40 stores. The transaction data is formatted analogous to a credit card statement, listing each transaction's date, store, and total basket amount.

This section is divided into four parts. We first describe the partner stores and how they differ on three key dimensions: category affinity, geographic affinity, and loyalty program policies. Then we describe the rich (exogenous) variation in reward rates that allows us to measure cross-reward effects across stores. Third we provide summary statistics that highlight the need for key features of the model developed in the next section. Finally, we lay out our hypotheses on cross-reward effects.

### **Partner stores**

The 40 focal stores are local branches of retailers with presence at the national level (i.e., with multiple branches in other cities across the country). We compare these stores on three dimensions that will be tied to our hypotheses of how the affinity between stores may affect reward spillovers. The dimensions are: categories sold, geographic proximity within the city, and the program policies of how consumers earn and redeem rewards at each store. Product categories sold across stores are grouped into ten sectors by the coalition: (1) health, optical, wellness, and beauty, (2) sport, (3) home and garden, (4) electronics, (5) travel (6) textiles and accessories, (7) restaurants and bars, (8) culture and education, (9) mobility, and (10) jewelry.

We measure *category proximity* across each pair of stores with the Jaccard similarity coefficient  $A_{jk}^{categ}$ , a measure commonly used for machine learning (e.g., Netzer et al. 2012). Let  $B_c^j$  equal 1 if store  $j$  sells category  $c$ , and 0 otherwise. Category proximity (i.e., Jaccard similarity) between stores  $j$  and  $k$  is calculated as shown in Equation 1



and ranges from zero to one.

$$A_{jk}^{categ} = \frac{\sum_c \min(B_c^j, B_c^k)}{\sum_c \max(B_c^j, B_c^k)} \quad (1)$$

Table 1 shows the frequency of category proximity across all pairs of focal stores. Stores that do not overlap in any category have the minimum category proximity of 0, regardless of how many categories each sells. Analogously, pairs of stores that compete in 100% of their categories have a category proximity equal to one. Most pairs (457) do not compete in any product category, 195 pairs of stores compete in some categories, and 128 pairs overlap in 100% of their categories.

Table 1: Frequency table of category proximity between all store pairs

$A_{jk}^{categ}$	0	0.2	0.25	0.33	0.5	1
Frequency	457	10	16	29	140	128

The extent to which two stores compete can also depend on physical proximity. On average, two stores are located only 3.2 kilometers apart, and the maximum observed distance is 10 kilometers. We measure geographic proximity between stores within the city as shown in Equation 2, where  $geo.dist_{jk}$  denotes geographic distance between two stores  $j$  and  $k$  and  $max.dist$  denotes the maximum distance. If two stores are in approximately the same location, the proximity measure equals 1. At the other extreme, the two stores that are the furthest apart in the city have a geographic proximity measure equal to zero. The histogram in Figure 1 shows a bi-modal distribution of geographic proximity across all pairs of partner stores in this city.

$$A_{jk}^{geo} = \frac{max.dist - geo.dist_{jk}}{max.dist} \quad (2)$$

[INSERT FIGURE 1 HERE]

Two aspects of the reward policies also differ across partner stores: how customers earn versus redeem points in the loyalty program. Differences in earning and redeeming policies across partners are a common feature in other popular European coalition programs (i.e., Nectar). The first policy, the level of a store’s reward tier, dictates how quickly customers earn points at a store. A store can offer customers either a low, medium, or high level of points earned per dollar<sup>2</sup>. The second policy dictates whether or not a customer can redeem rewards at a store. Periodically, the coalition mails vouchers to customers for the dollar value of their earned points. These vouchers are valid for two years at select partners. In this city, 33 out of the 40 stores accepted vouchers throughout the observation period, and seven never accepted them.

Conversations with the managers of this coalition program acknowledged that uncertain costs are the main reason why some partners do not accept vouchers. Stores are only reimbursed 90% of a voucher by the coalition operator. Thus, the impact of accepting vouchers on a store’s total margin depends on the percentage of the basket price that customers choose to save with vouchers. Consider a customer with a \$20 voucher that will thus cost a store \$2 to cash in with the loyalty program. A store with a 5% margin loses in the short run when the voucher is redeemed on a \$20 basket (\$1 margin), but makes a short-term profit when the voucher is redeemed on a larger \$60 basket (\$3 margin).

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<sup>2</sup>We use the term “dollar” instead of the currency observed to maintain the anonymity of the program’s location.

## Changes to reward policies

We now detail how reward policies were set and changed over time. The coalition’s managers set (1) the number of points offered at each of the three tier levels (low, medium, and high) and (2) the dollar value of a point. These policies affect every store in the coalition. Each franchise chooses (1) a redemption policy (yes / no) and (2) reward tier level (high / medium / low). The policies chosen by a franchise apply to all branches at the national level. Thus, the reward level at a particular store is not based on its sole performance, nor set by it.

There were nine dates at which reward rates changed at any of the 40 stores: 2007/01/05, 2007/01/25, 2007/02/01, 2008/09/30, 2009/09/01, 2009/10/07, 2009/11/20, 2009/12/07, and 2011/04/05. Table 2 shows the changing reward rates (i.e., the dollar value of points earned per \$100 dollars spent) offered by each store during each of the ten time epochs during which rates remained unchanged across stores. There were eight observed trajectories in which reward rates evolved across stores. Note that some partners experienced up to three changes in the rewards offered during the observation period (Trajectory 4).

Table 2: Pattern in the dollar value of rewards earned for \$100 spent at each store

Epoch start date	1	2	3	4	5	6	7	8	9	10	# stores
Trajectory 1	3.0	3.0	3.0	3.0	3.0	3.0	3.0	1.0	1.0	1.0	13
Trajectory 2	1.5	1.5	1.5	1.5	1.5	1.5	1.5	0.5	0.5	0.5	10
Trajectory 3	3.0	1.5	1.5	1.5	1.5	1.5	1.5	0.5	0.5	0.5	10
Trajectory 4	3.0	3.0	3.0	0.3	0.3	0.6	0.6	0.2	0.2	0.2	2
Trajectory 5	3.0	3.0	3.0	3.0	3.0	3.0	3.0	1.0	1.0	0.2	2
Trajectory 6	3.0	3.0	3.0	3.0	3.0	3.0	1.5	0.5	0.5	0.5	1
Trajectory 7	3.0	3.0	3.0	3.0	3.0	3.0	3.0	1.0	0.5	0.5	1
Trajectory 8	3.0	3.0	1.5	1.5	3.0	3.0	3.0	1.0	1.0	1.0	1

The changes to reward rates shown in Table 2 are of three types. The first type of change was a large points devaluation in November 2009 (Epoch 8). Points were

devalued to a third of their original value by the third-party coalition operator. One hundred points became equivalent to \$1 instead of \$3 for all trajectories. Second, two months before the devaluation (September 2009, Epoch 6), the coalition operator doubled the amount of earned points from purchases at all low tier stores from 0.1 points to 0.2 points per dollar spent. This tier-level change only applied to two of the partner stores in the city studied (Trajectory 4). The first and second types of changes were implemented by the third-party coalition operator to improve its margins from transactions across hundreds of partner stores across the country.

To compensate for the devaluation in November 2009, the coalition operator made it easier for customers to redeem rewards. The fungibility of vouchers was improved in three ways: (1) by mailing them more frequently (monthly instead of quarterly), (2) by increasing the denominations from only \$15 vouchers to \$5, \$10, \$20, \$50, and \$100 denominations, and (3) by allowing customers to redeem vouchers up to 100% of the basket price, instead of previously 30%. Although the changes to points and vouchers occurred at the same time, we can discern the effects of each because some partner stores accept vouchers while others do not.

The third type of change to reward rates are tier-level changes. Seventeen out of the forty partner stores were observed to switch tiers during the observation period. Ten stores changed from high to medium in January 2007 (Trajectory 3), two stores changed from high to low in February 2007 (Trajectory 4), two stores changed from high to low in April 2011 (Trajectory 5), one store changed from high to medium in October 2009 (Trajectory 6), one store changed from high to medium in December 2009, and the last store changed from high to medium in January 2007 and then switched back to high in September 2008 (Trajectory 8).

Tier-level changes to reward rates can be used to measure cross-reward effects on neighboring partner stores because these likely did not occur in response to anticipated

changes in local cross-reward effects in the city studied. First of all, these decisions were not made at the local level, but instead applied across all branches of a retailer across the country. Each store that experienced tier-level changes within the city belonged to one of nine retailers containing an average of nineteen branches across the country. Second, store managers do not observe reward spillovers since they do not have access to the coalition's data on how their customers purchase at other partners. Finally, managers are unlikely to have been able to anticipate any impact of potential changes in the reward spillovers from other partners in the city because their impact on store-level profitability was largely unknown before this study. The nature of this setting is similar to the context of Ozturk, Venkataraman and Chintagunta (2016), who assess competitive price reactions with the assumption that car dealership closures represent an exogenous shock to market structure.

### **Description of purchases**

The nature of the dataset (many stores and infrequent purchases) poses three main challenges to measure reward spillovers. First, purchases across stores are sparse. Customers tend to shop infrequently and across few partners due to the high-end nature of many of the stores. A large segment of customers (41%) only purchased at one of the forty partner stores during the observation period. Furthermore, those who did purchase at more than one store throughout the years rarely did so on the same day: only 2.3% of all daily purchase occasions included purchases at two or more of the top stores (Table 3). The first column in the table notes the purchase occasions in which we observe a customer purchasing in the city, but not at the top forty partner stores. The mean inter-purchase time of a customer across all partner stores in the city was 6.8 months.

Table 3: Number of top partners visited on a given daily purchase occasion

# top stores	0	1	2	3
# occasions	4250	11488	378	20

The second challenge to estimate cross-reward effects across each pair of stores is to parsimoniously accommodate a large number of stores (40 in our case) with sparse purchases between them. Figure 2 shows a highly skewed distribution of the number of purchases observed across the forty partner stores, portending our choice to estimate the model by sharing information across stores in a Bayesian manner (Gelman et al. 2003).

[INSERT FIGURE 2 HERE]

The third challenge is to capture a potential structural change in cross-reward effects after the large (1/3rd) points devaluation in November 2009. Table 4 compares two distributions before and after the devaluation. It shows the percentage of customers who are observed purchasing at  $n$  out of the 40 stores during the three years before the devaluation versus the three years after the devaluation. Although the number of customers in each of these periods is fairly similar, overall purchases decreased after the devaluation, from 8215 to 6634 throughout the coalition, and the percentage of customers co-patronizing three or more stores decreased from 35% to 25%.

Table 4: Percent of customers who purchased at  $n$  of the top stores

Three years	1 store	2 store	> 2 stores	customers	purchases
before the devaluation	44%	21%	35%	1025	8215
after the devaluation	52%	23%	25%	1073	6634

The model developed in the next section includes properties that overcome these challenges. First, the model statistically pools information across both customers and stores using Bayesian methods. Second, it uses latent links across the store network

to capture how cross-reward effects change depending on how similar stores are in each of three focal dimensions (category affinity, geographic proximity, and voucher policies). Third, it allows the nature of the cross-reward effects to structurally change after the one-third devaluation of the points currency. The model is also developed to test six hypotheses of how each store's rewards are expected to influence purchases at other partners.

## **Hypotheses**

We now present six hypotheses on how cross-reward effects may vary with three types of observables: redemption policies, store similarity, and the devaluation. The first hypothesis predicts cross-reward effects to be more favorable for stores that accept redemptions. Intuitively, higher rewards provide customers with more points available to spend, so voucher-accepting stores should benefit more from spillovers than those who do not.

*H1: Stores that accept vouchers receive more positive spillover effects from the reward rates of other stores.*

The following vignette motivates three predictions of how spillovers vary with store similarity in terms of categories sold and geographic proximity. Suppose that consumers are attracted to a store because they derive some utility from two of its attributes: location and categories offered. Points earned at the initial store may motivate a visit to earn more points at a different partner that is superior to the initial one on either dimension (better location or product categories). Therefore, on a second purchase occasion, a consumer is unlikely to derive higher utility from a "twin" store that is similar in both dimensions to the initial store, since the initial store was chosen over its twin at the first purchase occasion. Instead, a consumer that has become satiated for the initial product category may be inclined to visit a store

in the same (preferred) location that offers different (higher-utility) categories. A non-satiated consumer may be compelled to visit a competing store selling the same preferred product category but in a different location that happens to provide higher utility at the time of the second purchase occasion (i.e., closer to work or home).

This stylized example motivates three hypotheses of how spillovers vary with the geographic and category proximity between stores. We expect a positive main effect for geographic proximity, a positive main effect for category proximity, and a negative interaction between the two. Hypotheses 2 and 3 predict that stores that are similar in either category proximity or geographic location may potentially benefit from each other's rewards. Hypothesis 4 predicts a negative interaction between geographic and category affinity on how rewards influence purchases across stores.

*H2: Pairs of stores that compete in the same product categories experience more positive cross effects from each other.*

*H3: Stores receive more positive reward spillovers from nearby partners.*

*H4: Nearby stores receive more negative reward spillovers when these also compete in the same product categories, relative to nearby partners that do not sell the same product categories.*

The last two hypotheses predict how the two coalition-wide policy changes impacted reward spillovers. Recall the two changes in November 2009: points were devalued by a third while vouchers became easier to redeem. To evaluate the impact of the devaluation on cross effects, we directly compare the (estimated latent) links before versus after the event in Hypothesis 5. In contrast, Hypothesis 6 predicts a positive difference between the spillovers received by stores that accept vouchers vs. those that do not. While overall rewards decreased, the increase to the fungibility of vouchers made it easier for customers to redeem more frequently.



*H5: The points devaluation led to a decrease in the average magnitude of spillovers across stores.*

*H6: The increased fungibility of vouchers increased the difference between spillovers received by stores that accept voucher redemptions relative to spillovers received by stores that do not accept voucher redemptions.*

## A Network Affinity Model (NAM) for multi-store purchase incidence

We develop a model for customer purchases across the stores of a coalition to measure cross-reward effects. We observe customer  $i \in \{1, \dots, I\}$  shopping in a city on  $N_i$  purchase occasions. Let  $n$  denote her  $n$ th purchase occasion (i.e., a day purchasing at any partner in the city) and let  $t(n)$  denote the calendar time of purchase occasion  $n$ . The dependent variable  $y_{ijn}$  equals one if customer  $i$  purchased at a focal store  $j \in \{1, \dots, J = 40\}$  on the  $n$ th purchase occasion, and zero otherwise. The customer's net utility from purchasing at  $j$  versus not is modeled with a deterministic component  $V_{ijn}$  and an error term  $\epsilon_{ijn}$  that is independently drawn from a logistic distribution, leading to the common logit form for the propensity to purchase:  $p_{ijn} = \frac{e^{V_{ijn}}}{1+e^{V_{ijn}}}$ . We model  $V_{ijn}$  as a function of three elements: each customer's baseline store preferences  $\theta_{ij}$ , own-reward effects, and cross-reward effects (also referred to as *reward spillovers*). These three elements are analogous to the utility model of own and cross-price effects in multi-category choices within a store by Manchanda, Ansari and Gupta (1999). We will model own and cross-reward effects as functions of latent links between stores, an approach motivated by Trusov, Bodapati, and Bucklin (2010) who used latent links to determine influential customers in a social network.

Consider a network in which each node represents a partner store. Each pair of stores is connected through two links that indicate how reward rates at one store spill over to purchases of the other. Between two stores A and B there are two directed links,

analogous to a sender-receiver network (e.g., Stephen and Toubia 2010): A receives a spillover from B ( $\gamma_{B \rightarrow A}$ ) but also sends a spillover to B ( $\gamma_{A \rightarrow B}$ ). Links may vary across customers, over time, and also in magnitude and valence. Two positive links ( $\gamma_{A \rightarrow B} > 0$ ,  $\gamma_{B \rightarrow A} > 0$ ) correspond to the case where two stores mutually benefit from each other’s reward rates. A negative link in one direction ( $\gamma_{A \rightarrow B} < 0$ ) and a null link in the other direction ( $\gamma_{B \rightarrow A} \approx 0$ ) describes an asymmetric case when B is harmed by A’s rewards, while B’s rewards are not associated with A’s purchases.

Let  $\gamma_{k \rightarrow j, i, t(n)}$  denote the link strength “sent” from  $k$  to  $j$  (Equation 3). The link is modeled by multiplying  $k$ ’s (observed) reward rate with a weight  $\omega_{k \rightarrow j, i, d}$  that varies with voucher acceptance policies, store similarity (geographic and category proximity), and a time indicator  $d$ , where  $d = 0$  for transactions before  $t^*$  (November 2009), the time at which the coalition devalued points and increased voucher fungibility, and  $d = 1$  after that time. The specification allows us to test the hypotheses laid out in the previous section.

$$\gamma_{k \rightarrow j, i, t(n)} = \omega_{k \rightarrow j, i, d} R_{kt(n)} \quad (3)$$

$$\omega_{k \rightarrow j, i, d} = \begin{cases} \kappa_{0id}^{cross} + \kappa_{1id}^{cross} Vouch_j + \psi_{1id} A_{jk}^{categ} + \psi_{2id} A_{jk}^{geo} + \psi_{3id} A_{jk}^{categ} A_{jk}^{geo} & \text{if } k \neq j \\ \kappa_{0id}^{own} + \kappa_{1id}^{own} Vouch_j & \text{if } k = j \end{cases} \quad (4)$$

When  $k \neq j$ , a link represents spillovers across stores. The weight on others’ rewards includes a baseline level  $\kappa_{0id}^{cross}$ , a covariate  $Vouch_j = 1$  if  $j$  accepts voucher redemptions and zero otherwise, and three terms describing how spillovers may vary with category proximity  $A_{jk}^{categ}$  and geographic proximity  $A_{jk}^{geo}$ , which range between zero and one (Equations 1 and 2). The parameter  $\psi_1$  allows spillovers to vary with the category proximity between stores,  $\psi_2$  allows spillovers to vary with the geographic proximity within a city, and  $\psi_3$  denotes an interaction for stores that both sell similar products and are located nearby. Hypotheses 1-4 predict the valence of the cross-

effect parameters as follows:  $\kappa_1^{cross} > 0$ ,  $\psi_1 > 0$ ,  $\psi_2 > 0$ ,  $\psi_3 < 0$ . When  $k = j$ , links refer to own-reward effects, and so the weight  $\omega_{j \rightarrow j}$  excludes the last three terms of store similarity.

Allowing for links to change after the program-wide policy changes at  $t^*$  allows us to empirically test Hypotheses 5 and 6. Let the vector  $\lambda_{id}$  group the coefficients of own and cross-reward effects:  $\kappa_{0id}^{own}$ ,  $\kappa_{1id}^{own}$ ,  $\kappa_{0id}^{cross}$ ,  $\kappa_{1id}^{cross}$ ,  $\psi_{1id}$ ,  $\psi_{2id}$ ,  $\psi_{3id}$ . As shown in Equation 5,  $\lambda_{i0}$  is represented by  $\beta_i$ , and  $\delta$  is a vector of changes to own and cross-reward effects after  $t^*$ .

$$\lambda_{id} = \begin{cases} \beta_i & \text{if } d = 0 \\ \beta_i + \delta & \text{if } d = 1 \end{cases} \quad (5)$$

Having completed the description of a link, Equation 6 specifies how links enter the deterministic utility  $V_{ijn}$ . As previously mentioned, own effect is represented by a link sent from a store to itself:  $\gamma_{j \rightarrow j, i, t(n)}$ . The utility from rewards offered by other stores  $k \neq j$  are modeled as the average link received from partner stores. As shown in Equation 7, we can re-write the deterministic utility as a function of  $K = 7$  types of covariates.

$$V_{ijn} = \underbrace{\theta_{ij}}_{\text{baseline}} + \underbrace{\gamma_{j \rightarrow j, i, t(n)}}_{\text{own effect}} + \underbrace{E_{k \neq j}[\gamma_{k \rightarrow j, i, t(n)}]}_{\text{cross effects}} \quad (6)$$

$$= \theta_{ij} + X_{jt(n)} \lambda_{id} \quad (7)$$

$$\text{where } X_{jt(n)} = \left\{ R_{jt(n)}, R_{jt(n)} Vouch_j, \frac{\sum_{k \neq j} R_{kt(n)}}{J-1}, \frac{\sum_{k \neq j} R_{kt(n)} Vouch_j}{J-1}, \frac{\sum_{k \neq j} R_{kt(n)} A_{jk}^{Categ}}{J-1}, \frac{\sum_{k \neq j} R_{kt(n)} A_{jk}^{Geo}}{J-1}, \frac{\sum_{k \neq j} R_{kt(n)} A_{jk}^{Categ} A_{jk}^{Geo}}{J-1} \right\}$$

To summarize,  $V_{ijn}$  includes three types of parameters: baseline store preferences  $\theta_{ij}$ , pre-devaluation coefficients for the own and cross-reward effects  $\beta_i$ , and devaluation effects  $\delta$ . Consumer heterogeneity is accommodated by specifying the following distributions on  $\theta_i$  and  $\beta_i$ , where the vector  $\theta_i$  groups each customer's baseline store preferences  $\theta_{ij}$ .

$$\theta_i \sim MVN(\bar{\theta}, \Sigma_\theta)$$

$$\beta_i \sim MVN(\bar{\beta}, \Sigma_\beta)$$

The term  $\bar{\theta}$  represents the mean of each store's attractiveness,  $\Sigma_\theta$  represents the variance-covariance of attractiveness across stores,  $\bar{\beta}$  represents the mean across customers of the pre-devaluation coefficients for own and cross-reward effects, and  $\Sigma_\beta$  represents the variance-covariance across customers.

The Bayesian model is estimated using a Markov Chain Monte Carlo sampler. Appendix A contains more details of the estimation procedure as well as an example of parameter recovery for a simulated dataset. Additional simulation results are available upon request.

## Empirical Results

Our empirical results are organized around three major topics: model fit, hypothesis testing, and competitive inference. First, we use benchmarks to evaluate the empirical fit of our model and demonstrate the need and impact of various model components. Second, we use the model parameters to evaluate the hypotheses on reward spillovers. Third, we visualize the market structure of the partners through the spillover links.

### Model fit

The fit of our network affinity model (NAM) is compared with three nested benchmarks summarized in Table 5. The first benchmark CROSS excludes the own-effects from NAM. The other two benchmarks are called OWN and BSP. OWN includes individuals' baseline store preferences and own effects, but excludes cross effects. BSP, the most limited benchmark, only includes baseline store preferences and thus does not include any information on reward rates. Since OWN and BSP do not model cross effects, they are consistent with a null hypothesis of no reward spillovers across partner stores. Furthermore, these do not use information on the geographic and category affinity between stores.

Table 5: Summary of empirical models

	Deterministic utility $V_{ijn}$
NAM	$\theta_{ij} + \gamma_{j \rightarrow j, i, t(n)} + \sum_{k \neq j} \gamma_{k \rightarrow j, i, t(n)} / (N - 1)$
CROSS	$\theta_{ij} + \sum_{k \neq j} \gamma_{k \rightarrow j, i, t(n)} / (N - 1)$
OWN	$\theta_{ij} + \gamma_{j \rightarrow j, i, t(n)}$
BSP	$\theta_{ij}$

Table 6 compares the overall fit across the models with three statistics: the mean log likelihood (LL), the deviance information criterion (DIC) and the log-marginal density (LMD) (Spiegelhalter et al. 2002, Newton and Raftery 1994, Rossi, Allenby, and McCulloch 2005). NAM has the smallest magnitude for each of these standard measures relative to the benchmarks, indicating a better overall fit of the data.

Table 6: Measures of overall model fit

Model	Mean LL	DIC	LMD
NAM	-31085	48095	-31429
CROSS	-31292	48882	-31656
OWN	-31531	49675	-31849
BSP	-31821	50628	-32141

Table 7 compares fit using posterior predictive checks (PPC) of particular interest to cross-store purchasing. The PPCs results contain the posterior means of error statistics calculated from 1000 datasets simulated from the posterior distributions of model parameters (Gelman et al. 2003). The first PPC is the sum of squared errors (SSE) between the actual and predicted number of transactions that each customer made at each store. The last two PPCs are the sum of squared errors between the actual and expected number of customers who co-patronized each *pair* of stores before vs. after November 2009. A smaller magnitude indicates a better fit for each of the measures. The sum of squared errors for NAM are the lowest compared to the benchmarks, marking an improvement in fit in terms of the number of transactions per store and cross-store patronage. These PPCs on co-store purchasing measures provide a more rigorous assessment of model fit and of how NAM provides improvement over simpler models.

Table 7: SSE between observed and predicted patronage for single and pairs of stores

Sum of squared errors (SSE)	NAM	CROSS	OWN	BSP
# transactions	16806	16906	17069	17298
cross-store patronage (pre-deval)	9560	10098	10938	11274
cross-store patronage (post-deval)	7061	7636	8099	8747

*Evidence for the proximity hypotheses*

We now analyze the cross-reward effects estimated by NAM, which are represented by the latent spillover links across the partner network. First we test our hypotheses at the population-level (across customers), and then for individual customers. Table 8 shows the posterior distributions of the population-level parameters for own and cross-reward effects. Recall that Hypothesis 1 predicts that stores that accept voucher redemptions should receive more positive (or less negative) cross-reward effects. Hypothesis 2 is supported if spillovers tend to be more positive between pairs of stores that overlap in categories sold, Hypothesis 3 is supported if spillovers tend

to be more positive between partners that are geographically close, and Hypothesis 4 predicts a negative interaction between nearby stores that are also competitors. Also recall that Hypotheses 1-4 predict the valence of the cross-effect parameters as follows:  $\kappa_1^{cross} > 0$ ,  $\psi_1 > 0$ ,  $\psi_2 > 0$ ,  $\psi_3 < 0$ .

Table 8: Population-level parameters  $\bar{\beta}$  and  $\bar{\beta} + \bar{\delta}$

Before Devaluation			After Devaluation		
Par.	Mean	(2.5%, 95.7%)	Par.	Mean	(2.5%, 97.5%)
$\bar{\kappa}_0^{own}$	-0.04	(-0.07, 0.16)	$\bar{\kappa}_0^{own} + \delta_{\kappa_0^{own}}$	0.14	(-0.07, 0.35)
$\bar{\kappa}_1^{own}$	-0.23	(-0.39, -0.07)	$\bar{\kappa}_1^{own} + \delta_{\kappa_1^{own}}$	0.20	(-0.14, 0.59)
$\bar{\kappa}_0^{cross}$	-1.95	(-2.50, -1.31)	$\bar{\kappa}_0^{cross} + \delta_{\kappa_0^{cross}}$	-2.23	(-2.99, -1.44)
$\bar{\kappa}_1^{cross}$	0.59	(0.36, 0.85)	$\bar{\kappa}_1^{cross} + \delta_{\kappa_1^{cross}}$	0.08	(-0.44, 0.62)
$\bar{\psi}_1$	2.76	(1.82, 3.90)	$\bar{\psi}_1 + \delta_{\psi_1}$	2.76	(1.72, 3.93)
$\bar{\psi}_2$	1.81	(1.04, 2.39)	$\bar{\psi}_2 + \delta_{\psi_2}$	1.78	(0.85, 2.51)
$\bar{\psi}_3$	-3.50	(-4.92, -2.22)	$\bar{\psi}_3 + \delta_{\psi_3}$	-3.42	(-4.87, -2.04)

First we examine evidence before the devaluation. The baseline cross-reward effect is negative ( $\bar{\kappa}_0^{cross} = -1.95$ , with the 95% posterior interval not containing zero), implying that increases in reward rates at one partner store tend to be associated with a decrease in purchases at other partners. Consistent with Hypothesis 1, stores that accept redemptions have a less negative baseline level of spillovers received:  $\bar{\kappa}_1^{cross}$  is positive (0.59) and significantly different from zero. Thus, our first finding is that stores that offer more generous reward policies (in particular, those that accept voucher redemptions) benefit more from other's rewards than those that do not.

Hypotheses 2-4 are also supported by the population-level parameters in Table 8: the posterior means of  $\bar{\psi}_1$  and  $\bar{\psi}_2$  are both significantly positive,  $\bar{\psi}_3$  is negative, and zero is excluded from the respective 95% posterior intervals. Interestingly, the negative magnitude of  $\bar{\psi}_3$  (-3.50) is less than  $\bar{\psi}_1 + \bar{\psi}_2$  (4.6), suggesting that while stores that are more similar on either dimension (categories or location) are more likely to benefit from each other's reward rates, this advantage is smaller for stores very similar in

both dimensions. This finding implies that the coalition may be able to better accommodate close category competitors when these are not geographically close within a city.

To complement the statistical support for Hypotheses 2-4, we plot the distributions of spillovers received from partners that are far versus close in terms of category proximity (Figure 3 ) and geographic proximity (Figure 4). These links are calculated using the reward rates before the devaluation, and are also weighted by each customer’s total purchases throughout the observation period. Figure 3 shows that positive spillovers are more commonly received from stores selling the same product categories: the percentage of positive links received are 36.0% from stores with no category overlap, 43.6% from stores with some overlap, and 54.3% from stores with full overlap. Similarly, Figure 4 suggests that stores in popular locations seem to benefit the most from the coalition.

[INSERT FIGURE 3 HERE]

[INSERT FIGURE 4 HERE]

A comparison of how spillovers changed after the devaluation provides additional insight into the value of a shared reward currency. The one-third devaluation is similar to a situation in which the currency would become “worthless,” in which case there would be no coalition program. First, Hypothesis 1 is no longer supported, since  $\bar{\kappa}_1^{cross} + \delta_{\kappa_1^{cross}}$  is not significantly different from zero. Thus, despite the increase in voucher fungibility, the devaluation eroded the advantage of accepting vouchers. At the population level, the post-devaluation parameters are still consistent and significant with Hypotheses 2-4.

While at the individual level, the effects for Hypotheses 1-4 are consistent with the population-level results, there is significant heterogeneity across customers. Table 9



shows the percentage of consumers with individual-level posterior means consistent with each hypothesis, as well as the percentage of consumers for which each hypothesis is supported at the 95% level. While most individuals (over 90%) have individual-level parameters consistent with the population-level results (e.g. positive individual-level posterior mean when the population-level effect is predicted to be positive), zero is included in the individual-level posterior intervals for nearly all customers due to the low number of observations per person, demonstrating the value of the “meta-analysis” in our Bayesian model.

Table 9: Individual-level support of hypotheses

Hypothesis	Before devaluation		After devaluation	
	Consistent	Significant	Consistent	Significant
1	94.0%	1.0%	64.7%	0.1%
2	98.6%	2.2%	98.7%	2.3%
3	90.6%	0.6%	90.6%	0.6%
4	99.7%	2.0%	99.6%	1.6%

We now show evidence that the devaluation weakened cross-reward effects across all stores, regardless of their voucher policies. Hypotheses 5 and 6 predict how cross-reward effects were affected by the coalition’s policy changes to both the value of rewards and voucher fungibility. To evaluate both of these hypotheses, we compare the posterior distribution of the links before and after the devaluation. Hypothesis 5 is supported if the average magnitude of spillovers decreased after the devaluation. We test this hypothesis with the statistic  $s_{H5}$ , where the expectation is an average over all customers  $i$ , focal stores  $j$ , and other stores  $k$ . Consistent with the hypothesis, the posterior mean for  $s_{H5}$  is positive (1.93) and the 95% posterior interval excludes zero (1.46, 2.50). At the individual level, Hypothesis 5 is supported and significant for 99.9% of customers.

$$s_{H5} = E[|\gamma_{k \rightarrow j, i, t=t^*-\epsilon}| - |\gamma_{k \rightarrow j, i, t=t^*+\epsilon}|] \quad (8)$$

To test Hypothesis 6 we calculate the *difference* in spillovers for stores that accept vs. do not accept vouchers. Hypothesis 6 is supported if this difference increased after the increase in voucher fungibility. We test this hypothesis with the statistic  $s_{H6}$ , which we expect to be negative. While the posterior mean for  $s_{H6}$  is negative (-0.04), the magnitude is low and the 95% posterior interval includes zero (-0.44, 0.34). Analogously, at the individual level, Hypothesis 6 is significantly supported for only 0.7% of customers.

$$s_{H6} = \left( E_{\{j:V_j=1\}}[\gamma_{k \rightarrow j,i,t=t^*-\epsilon}] - E_{\{j:V_j=0\}}[\gamma_{k \rightarrow j,i,t=t^*-\epsilon}] \right) - \left( E_{\{j:V_j=1\}}[\gamma_{k \rightarrow j,i,t=t^*+\epsilon}] - E_{\{j:V_j=0\}}[\gamma_{k \rightarrow j,i,t=t^*+\epsilon}] \right) \quad (9)$$

Our analysis of the parameters of the cross-reward effects has shown that category competition can be offset through geographical distance in a coalition program. Furthermore, in a program with a strong reward currency, more generous store-level policies (higher reward rates and accepting redemptions) can help partners to compete through differentiation. Finally, we have also found evidence that the value of the points currency does influence how customers purchase across stores. The final part of this section further investigates the strategic implications of this result.

#### *Market structure based on reward cross effects*

The spillover links estimated by NAM can be used in multiple ways to develop a strategy to improve the management of the coalition reward program. This section leverages the links to segment customers, visualize the market structure of the partners, and evaluate the effects of a counterfactual policy change. First, we segment customers into three groups using K-means clustering on their spillover links before the devaluation. Segment 1 has 1148 customers, segment 2 has 307 customers, and segment 3 has 181 customers. The mean purchases per customer are 6.4 purchases

for segment 1, 9.3 purchases for segment 2, and 11.6 purchases for segment 3. Thus, while segment 3 is the smallest group, its customers are the most avid purchasers.

The spillovers for customers in each segment are visualized using heat maps. Figures 5, 6 and 7 relate diverse types of stores through a common purchase-based metric: the spillover links. From a business intelligence perspective, such heat maps provide the coalition program, and each partner store, an assessment of near versus far stores from a competitive perspective. Each heat map is composed of squares. Each square represents the affinity link sent by the column store to the row store. Each heat map illustrates both positive and negative tensions between partner reward rates. Red squares denote negative spillovers and blue ones denote positive spillovers. The scales are asymmetric: the red-to-white scale ranges from -4.7 to zero and white-to-blue scale ranges from zero to 3.0. Partner stores are ordered from most (store 1) to least (store 40) observed transactions.

[INSERT FIGURE 5 HERE]

[INSERT FIGURE 6 HERE]

[INSERT FIGURE 7 HERE]

Each segment-specific heat map can be used to compare the degree of asymmetry in the influence or “power” between specific pairs of partner stores, which can be a key tool for the coalition managers to better understand and advise their partners on their activities within the program. First, consider how store 1 influences other partners through the high-value customers of segment 3 (Figure 7): store 1 sends strong positive spillovers to partners 9 and 19, and strong negative spillovers to store 2, but in exchange, these stores exert very little influence on store 1 (i.e., the links received by store 1 are weak). Second, within each heat map we can also identify “synergistic groups” of partners likely to mutually benefit from additional cross-marketing

opportunities (i.e., joint targeted promotions), such as stores 20, 21, and 22 in Figure 7.

Third, since the reward spillover “landscape” varies considerably across customers, the heat maps are also an effective tool for the coalition operators to communicate to partner stores how the value of the program varies *across different customer segments*. For example, store 27 sends negative spillovers to partners 3, 4, 6 and 7 for customers in segments 1 and 2 (Figures 5 and 6), but it instead sends positive spillovers to these stores for the most valuable group, segment 3 (Figure 7).

Having visualized the spillovers for each pair of stores, we now aggregate the links across the pairs to identify which stores influence others and which are most influenced by others’ rewards. Our approach is based on Kamakura and Russell (1989) which summarized negative cross-price elasticities among brands. Our two metrics of coalition influence, called *vulnerability* and *competitive clout*, are shown in Equations 10 and 11 (for ease of illustration, these equations suppress customer and time indices of the affinity links).

$$Clout_j = \sum_{k \neq j} |\gamma_{j \rightarrow k}| \quad (10)$$

$$Vulnerability_j = \sum_{k \neq j} |\gamma_{k \rightarrow j}| \quad (11)$$

Vulnerability captures how sensitive a store is to other’s reward rates and competitive clout captures how influential the reward rates of a store are on other partners. Competitive clout sums over the absolute value of the affinity links “sent” by a store to other’s in the network, and vulnerability sums the absolute value of the affinity links “received” by a store from the rewards of other partners<sup>3</sup>. The metrics are calculated

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<sup>3</sup>Using the absolute value allows both positive and negative spillovers to factor into the clout and vulnerability measures of how strongly firms influence each other without cancelling each other out.

with a weighted average of the links from each customer’s posterior draws, weighted in proportion to each customer’s observed purchases.

We visualize the clout and vulnerability metrics under three real and hypothetical scenarios: (1) before the devaluation, (2) after the devaluation and (3) under a counterfactual scenario. The diagonal lines in each map in Figure 8 mark where clout is equal to vulnerability.

[INSERT FIGURE 8 HERE]

The first of the three plots in Figure 8 shows the market structure before the devaluation. In it emerges a group of highly influential stores (i.e., those in the top-left corner, above the diagonal) with more clout than vulnerability. Partners in the bottom-right corner of the plot are “underdogs” with more vulnerability than clout.

The second plot in Figure 8 shows the market structure after the devaluation: stores are aligned closer to the diagonal, where they influence about as much as they are influenced by others. Furthermore, they are less able to differentiate themselves through their policies, perhaps explaining why three stores chose to downgrade their reward tier after the devaluation. Thus, the actual measures taken to compensate for the devaluation (i.e., increasing the fungibility of points and doubling points at low-tier partners) were not effective in maintaining the ability of partners to differentiate themselves through their reward policies. We propose a customer-centric targeting strategy to more effectively “combat” these negative consequences of the devaluation.

The third plot in Figure 8 shows the market structure under a hypothetical customer-centric coalition program. In our counterfactual, we suppose that after the devaluation, high-value customers are selected to join a gold tier in which they can earn points three times as fast, thus effectively maintaining the pre-devaluation reward rates for this select group. To construct a customer-centric gold tier, we selected

the top five customers who made the most purchases at each partner, resulting in a group of 155 unique customers. This counterfactual disregards any differences in behavior that would arise from customers aspiring to join the gold tier. Compared to the post-devaluation positioning plot, the third plot in Figure 8 shows that the dispersion of clout and vulnerability across stores is significantly restored with the program. Thus, the results suggest that compensating for the points devaluation with a customer-centric program (that includes a higher-earning gold tier) can be an effective strategy to restore the ability and incentives of partner stores to differentiate from one another through their two reward policy levers: store-level reward tiers and voucher acceptance. In addition, if customer-level metrics from this gold tier were made available to the coalition partners, the coalition loyalty program may be able to restore its value even further to this targeted population.

## General Discussion

This paper has utilized a unique dataset from a coalition loyalty program to assess how shared points across different types of retail stores influence how customers purchase across them. Cross-reward effects were measured and empirically related to firms' reward policies and to the affinity between partner stores. These cross-reward effects were used to visualize the market structure of the diverse set of partner stores in two ways: one which illustrated asymmetric relationships between each dyad, and another which compared aggregate store-level metrics of influence (competitive clout and vulnerability). These maps are useful for managers seeking to comprehend nuances in the tensions and synergies across partner stores, monitor their evolution, and to identify partners that may potentially benefit from cross-marketing opportunities.

We first found evidence showing that store affinity, in terms of categories sold or geographic proximity, can foster positive cross-reward effects. However, these syner-

gies are attenuated for nearby competitors: partners that are simultaneously similar in both dimensions. Before the devaluation, stores that did not accept redemptions received more negative spillovers from other partners.

Second, we compared cross-reward effects before versus after the one-third devaluation as a “proxy” for what would happen if these stores were to dissolve their partnership. Before the devaluation we observed both positive and negative cross-reward effects across pairs of stores. The devaluation not only lowered the magnitude of cross-reward effects, but also led to a more homogeneous and competitive landscape of negative, low-magnitude reward spillovers, perhaps potentially leading to increased competition among the stores on other dimensions.

Third, we simulated a hypothetical scenario with a customer-centric targeting strategy to “combat” the negative consequences of the devaluation. We simulated a scenario in which high-value customers were selected into a gold tier status in which they earn points at the pre-devaluation reward levels. The key takeaway from this hypothetical exercise is that offering higher reward rates, even for a select subset of customers, maintains the incentives for firms to offer better reward policies, by enabling them to benefit by differentiating themselves from other partners through the two key reward levers of the coalition loyalty program.

Despite the richness of the dataset, it imposed several limitations to our findings. First, since we do not observe changes in the composition of the coalition, simulations of changes in the impact of the store portfolio may suffer from the Lucas critique (Lucas 1976). Second, we did not explicitly model redemptions because they are not reliably marked in the dataset, based on conversations with the coalition managers. Third, this setting does not include online purchases. While cross-reward effects in our setting are strongly tied to the geographic proximity within a city of partner stores, it is still unknown whether the old adage of “location is everything” will continue to

hold with the exponential rise of online sales (Bell 2014).

Despite the limitations, our findings invite multiple avenues for future research. The rise of coalition programs are part of a larger trend for retailers to adopt new types of reward programs that are structured in a fundamentally different way from traditional single store programs. In particular, we are observing a proliferation of reward programs in which retailers do not explicitly incentivize customers to return multiple times in order to “cash-in” rewards (Stourm, Bradlow, Fader 2015). As a consequence, points in these programs do not impose future switching costs on consumers. Rewards in these programs are more fungible and thus more similar to cash than those offered in classic reward schemes, such as a “buy 10 meals, get one free” scheme.

Future research can investigate the implications of other innovations in reward programs that may further increase the fungibility of rewards. First, the digitalization of rewards eases the exchange of information among parties. Mileage-tracking websites are one prominent example which allow customers to keep track of various loyalty programs in one place, and even allow customers to more easily compare the value of points across competing airline carriers, hotels, and car rental companies (McCartney 2011).

Second, the existence of increasing competition pressures companies to match rewards earned at competitors. Many hotel chains and airline carriers have status matching programs that reward customers with special status if they have earned a similar status at a competitor. One example is the “Status Match . . . No Catch” policy at Best Western Hotels & Resorts which matches a customer’s elite status in any other hotel loyalty program, free of charge. These programs make single-firm programs similar to coalition programs, in which a customer can earn a special status valid across all partners even though the bulk of his spending was directed at a few firms.



Third, firms are facing pressure to adapt their programs to leverage mobile technologies. By doing so, rewards have the potential to become truly redeemable “anytime, anyplace.” Finally, competitive pressures also encourage firms to increasingly allow customers to more easily transfer points to other customers. In summary, reward programs are rapidly evolving and may have an increasingly important impact on store visitation, prices, and promotions, all aspects of particular salience to marketing practitioners and scholars.

## APPENDIX A

The appendix details the Bayesian specification of the model, describes the estimation procedure, and illustrates parameter recovery. First we complete the Bayesian specification of the Network Affinity Model (NAM) with conjugate hyperprior distributions.

$$\bar{\theta} \sim MVN(\theta_0, \Omega_\theta)$$

$$\Sigma_\theta \sim IW(\nu_\theta, \Delta_\theta)$$

$$\bar{\beta} \sim MVN(\beta_0, \Omega_\beta)$$

$$\Sigma_\beta \sim IW(\nu_\beta, \Delta_\beta)$$

$$\bar{\delta} \sim MVN(\delta_0, \Omega_\delta)$$

The hyperprior means  $\theta_0$ ,  $\beta_0$ , and  $\delta_0$  are set to zero. The remaining parameters are set as follows, where  $K = 7$  is the dimension of  $\bar{\beta}$  and  $J = 40$  is the dimension of  $\bar{\theta}$ :  $\nu_\theta = J$ ,  $\Delta_\theta = \Omega_\theta = JI_J$ ,  $\nu_\beta = K$ ,  $\Delta_\beta = \Omega_\beta = KI_K$ ,  $\Omega_\delta = I_k 0.05$ .

The estimation procedure is a Markov Chain Monte Carlo sampler coded in the R software. Parameters entering the likelihood  $(\theta_{ij}, \beta_i, \delta)$  are sequentially sampled from their conditional posterior distributions using a random-walk Metropolis sampler. The step sizes for each of these parameters are adapted during the first 20,000 iterations to maintain acceptance rates between 30% and 40%. Parameters governing the prior distributions  $(\bar{\theta}, \Sigma_\theta, \bar{\beta}, \text{ and } \Sigma_\beta)$  can be directly sampled from their marginal posterior distributions due to their closed-form marginal posterior distributions.

We estimated the model with three independent chains of 400,000 iterations, and kept the last 150,000 draws of each. We thinned every 50 draws to reduce autocorrelation. Convergence was determined using the Gelman and Rubin (1992) diagnostic

of between-to-within chain variance.

We illustrate parameter recovery by estimating the network affinity model on a simulated dataset. We used the observed covariates from the 40 stores to simulate a new set of purchase choices for 200 individuals. We ran a single chain for 400,000 iterations and kept every 50 draws to reduce autocorrelation. The last half were used to compare the posterior distributions to the actual values. Table 10 compares the true values, the estimated values, and the 95% posterior intervals of the population-level parameters  $\bar{\beta}$  and  $\delta$ . The true values of each of the forty elements in  $\bar{\theta}$  were set to -2, and each of these true values were also contained within their 95% posterior intervals.

Table 10: Parameter recovery for Affinity Model

Parameter	Actual	Estimated	95% PI
$\bar{\kappa}_0$	0.5	0.51	(0.47, 0.55)
$\bar{\kappa}_1$	-1.0	-1.00	(-1.06, -0.95)
$\psi_0$	-3.0	-2.95	(-3.05, -2.81)
$\psi_1$	0.5	0.50	(0.43, 0.56)
$\psi_2$	2.0	1.98	(1.76, 2.17)
$\psi_3$	2.0	1.96	(1.82, 2.08)
$\psi_4$	-2.0	-1.95	(-2.19, -1.72)
$\delta_{\kappa_0}$	0.1	0.12	(0.09, 0.15)
$\delta_{\kappa_1}$	-0.1	-0.13	(-0.18, -0.08)
$\delta_{\psi_0}$	-1.0	-0.91	(-1.10, -0.72)
$\delta_{\psi_1}$	-1.0	-1.04	(-1.14, -0.92)
$\delta_{\psi_2}$	-0.1	-0.07	(-0.32, 0.21)
$\delta_{\psi_3}$	-0.4	-0.45	(-0.66, -0.25)
$\delta_{\psi_4}$	0.0	-0.07	(-0.37, 0.23)

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## FIGURES

Figure 1: Histogram of geographic proximity among pairs of stores

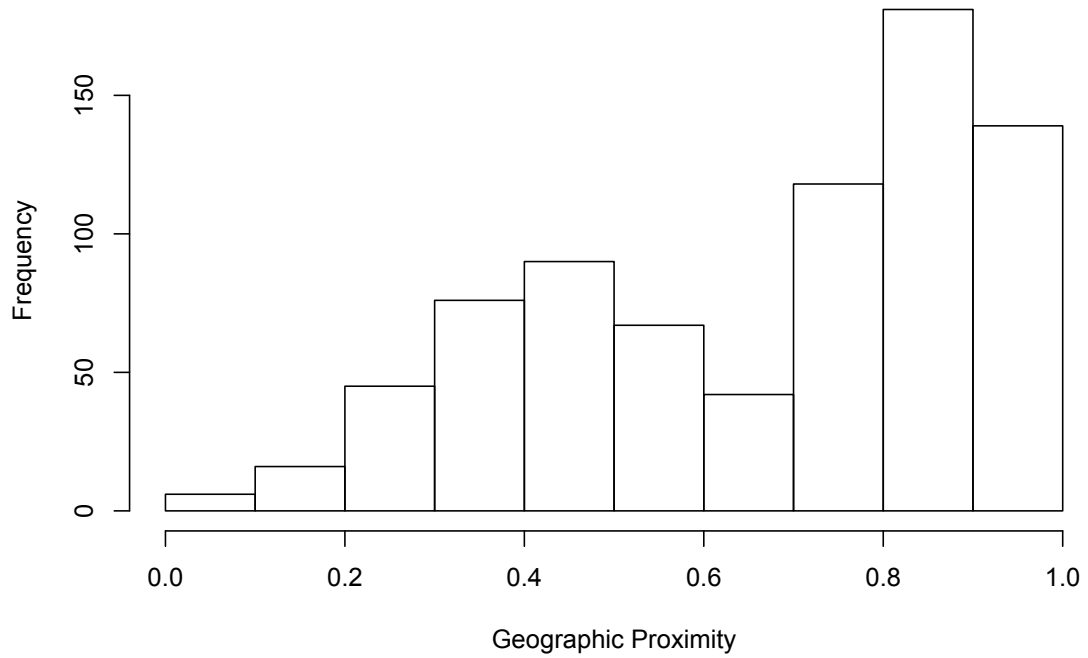


Figure 2: Distribution of purchases observed at each store

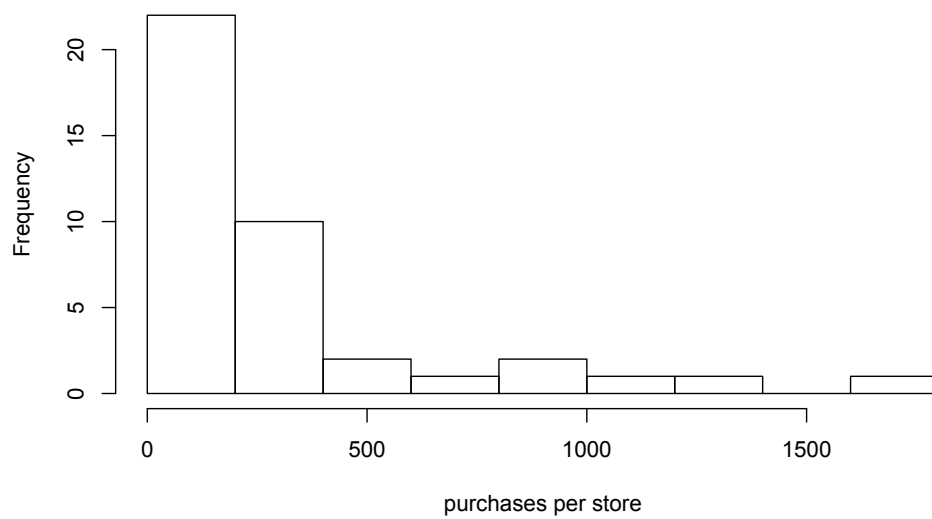


Figure 3: Distribution of links received by category proximity

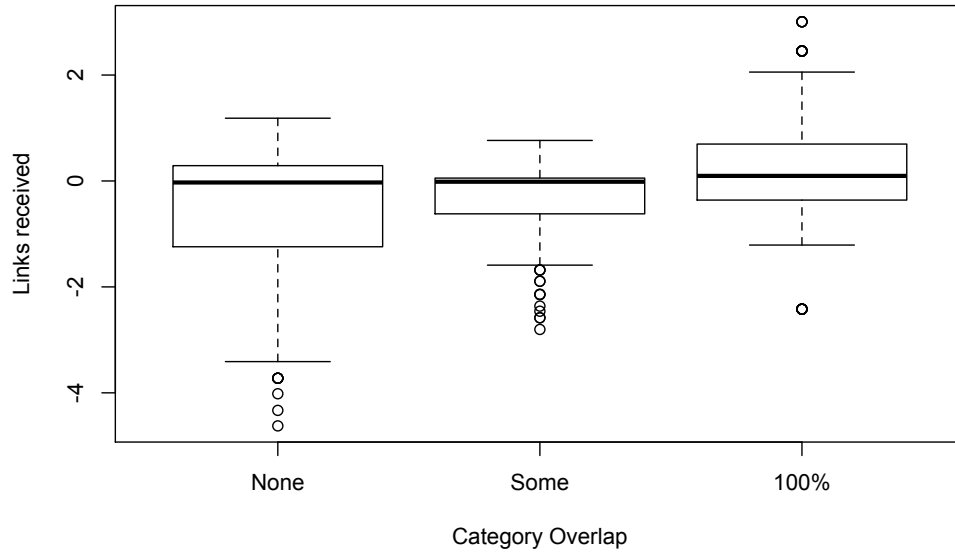


Figure 4: Distribution of links received by geographic proximity

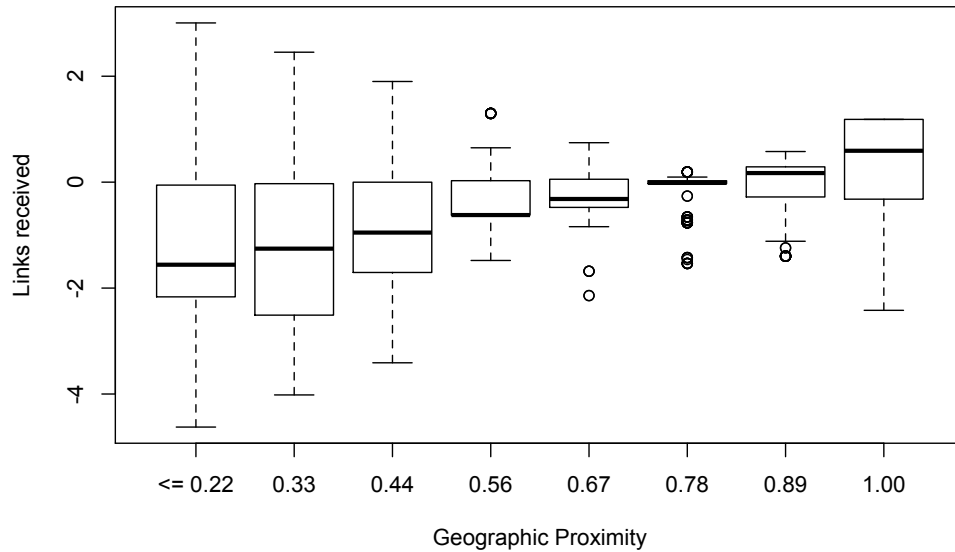


Figure 5: Reward spillovers across store pairs: heat map for segment 1 customers (the least avid purchasers)

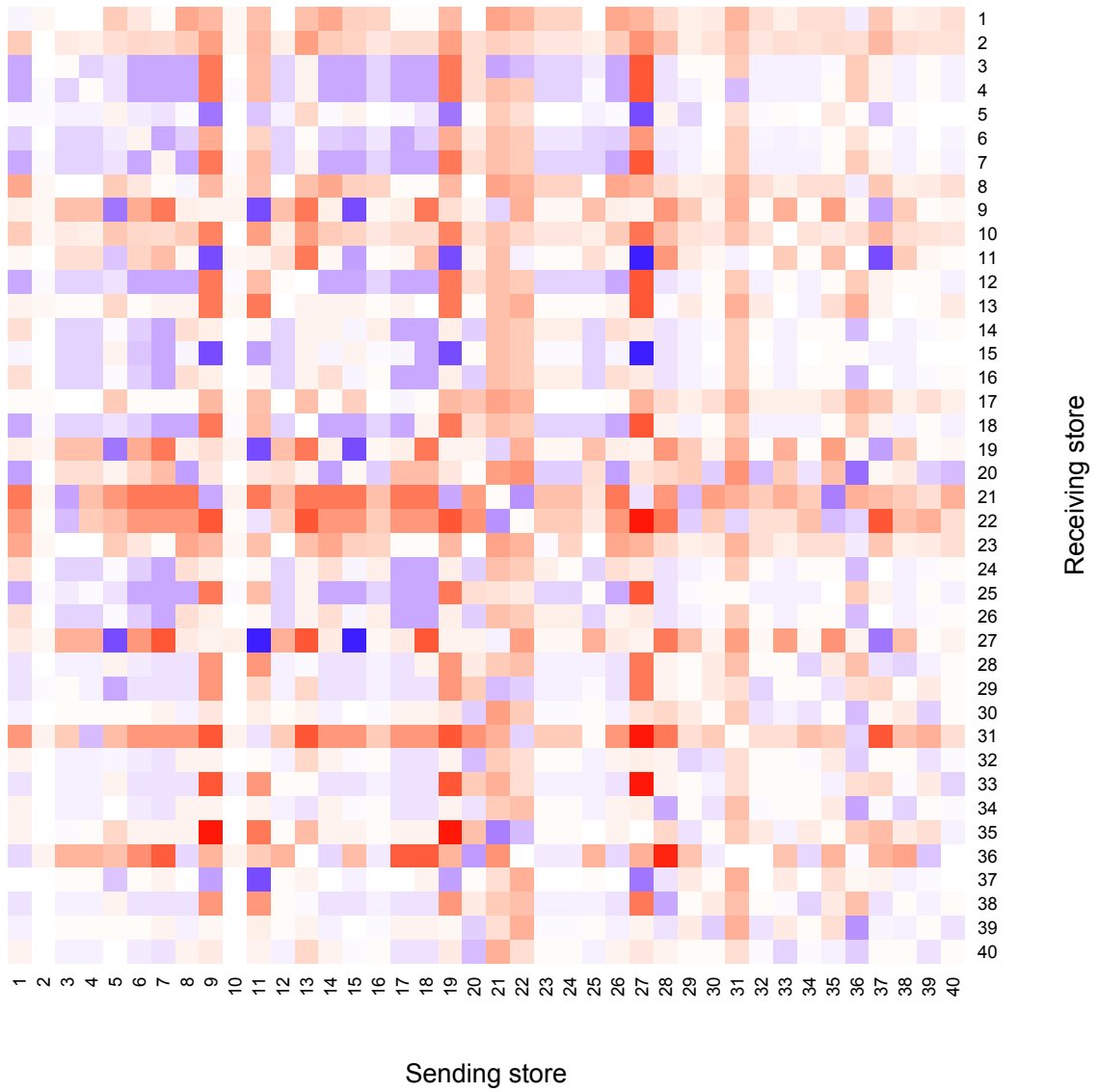




Figure 6: Reward spillovers across store pairs: heat map for segment 2 customers (the second-most avid purchasers)

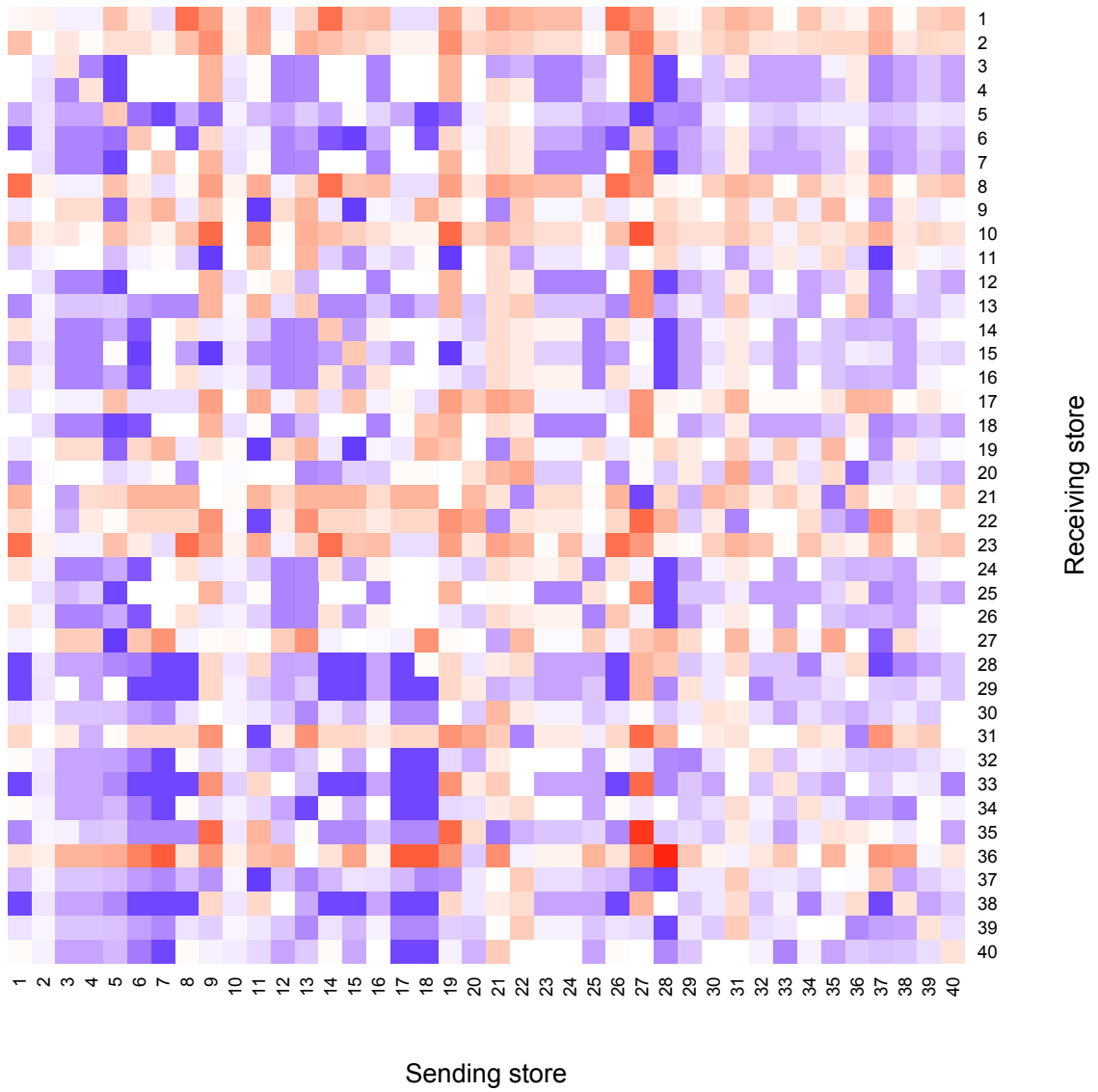


Figure 7: Reward spillovers across store pairs: heat map for segment 3 customers (the most avid purchasers)

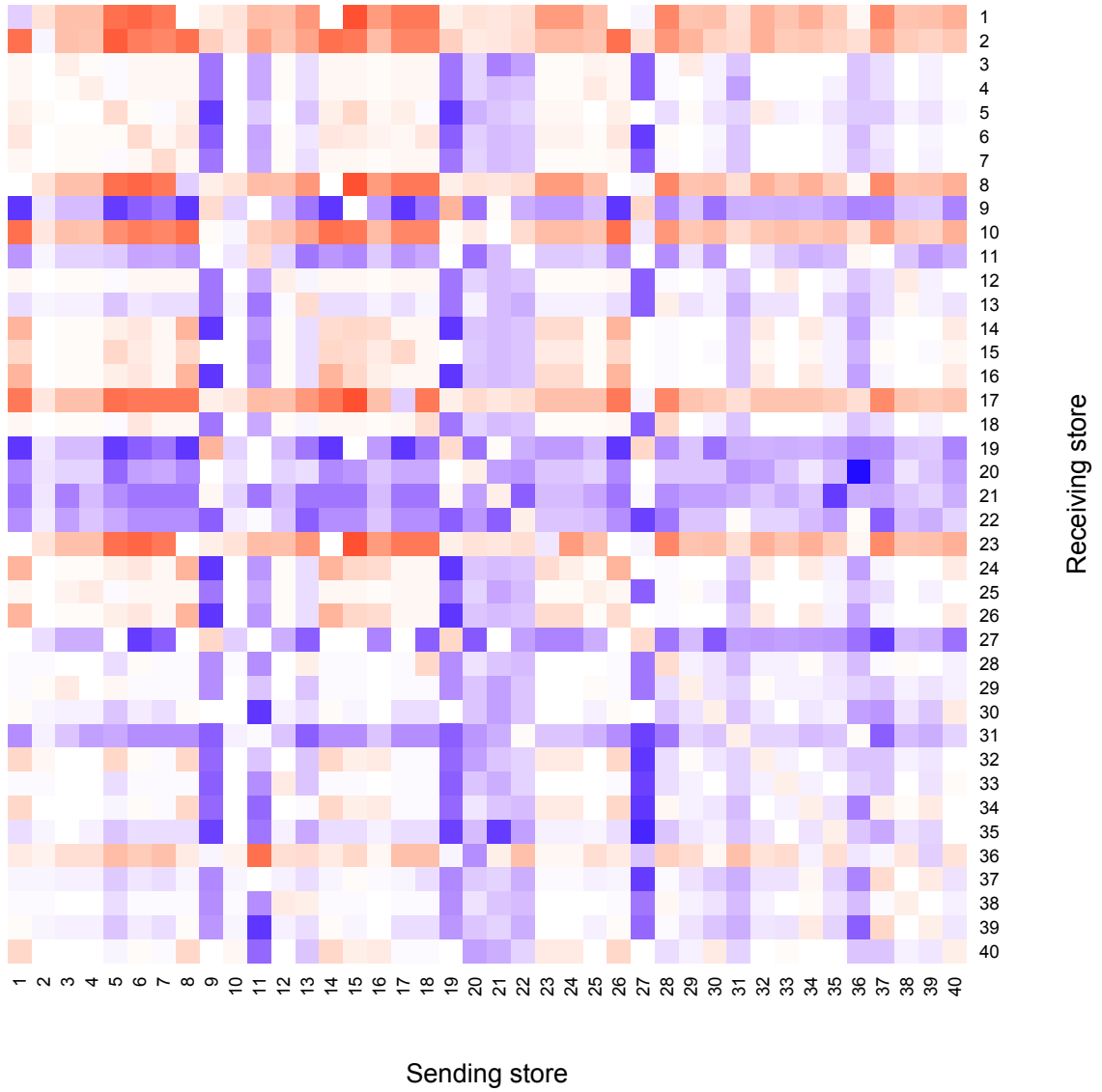


Figure 8: Vulnerability vs. Competitive Clout of Reward Spillovers

